Academic subject: Advanced Geometry 2				
Degree Class: LM-40 – Matematica		**Degree Course:** Mathematics	**Academic Year:** 2018/2019	
		Kind of class: Mandatory/Optional depending on the Curriculum	**Year:**	**Period:** 2
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115				
Language: Italian	**Compulsory Attendance:** no			
Subject Teacher: Antonio Lotta	**Tel:** +390805442656 **e–mail:** antonio.lotta@uniba.it	**Office:** Department of Mathematics Room 7, Floor II	**Office days and hours:** By appointment	
Prerequisites: Basic knowledge of smooth manifolds and Lie groups. Elementary notions about Riemannian metrics: Levi-Civita connection, geodesics, curvature.				
Educational objectives: Acquiring knowledge of some advanced topics of modern Riemannian Geometry, especially concerning Riemannian submersions, homogeneous and symmetric spaces and some results concerning the relationship between curvature and topology, providing the necessary background for further study of the subject at Phd level.				
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring some fundamental concepts and proof techniques in modern differential geometry. **Applying knowledge and understanding:** The acquired theoretical knowledge is useful in great part of mathematics and of theoretical physics. **Making judgements:** Ability to comprehend and rework the proofs of meaningful mathematical results. Ability to test some general facts on specific examples. **Communication:** Students should acquire the mathematical language and formalism necessary to read and comprehend advanced textbooks and specialized literature on the subject and to explain the acquired knowledge. **Lifelong learning skills:** Acquiring suitable learning methods, supported by text consultation and by elaborating on questions periodically suggested during the course.			

Course program

Riemannian submersions. Submersions between smooth manifolds. The vertical distribution associated with a submersion. Riemannian submersions. Horizontal, basic vector fields and the horizontal distribution. The fundamental tensor fields: definition, properties and geometric meaning. Conditions for the integrability of the horizontal distribution. Formulas relating the curvatures of the total space, of the base space and of the fibers. Vertical, horizontal, mixed sectional curvatures.

Examples of Riemannian submersions. The projection of a Riemannian covering. Warped product metrics; the projection of a warped product space onto the first factor. The Sasaki metric on the tangent bundle of a Riemannian manifold and the natural projection. Description of the fundamental vector fields associated with each of these types of submersions.

Geodesics and submersions. A characterization of the geodesics on the total space of a Riemannian submersion. Horiziontal geodesics. Clairaut submersions. The Bishop theorem. Examples.

Homogeneous spaces. Smooth actions of Lie groups on manifolds. Fundamental vector fields. Quotient of a manifold by a regular equivalence relation: theorem of Godement. The exponential map of a Lie group. The closed subgroup theorem. Quotient of a Lie group by a closed subgroup. Examples.

Riemannian homogeneous spaces. Homogeneous Riemannian manifolds. Examples. Isotropy representation. Criterion for the existence of invariant metrics on a homogeneous space. Existence of bi-invariant metrics on compact Lie groups. Homogeneous spaces with irreducible isotropy representation. The Levi-Civita connection of an invariant metric in the reductive case. Killing fields and their characterization. Nomizu's formula for the curvature tensor of a reductive homogeneous Riemannian manifold. Naturally reductive homogeneous spaces. Normal metrics, Samelson's theorem. Invariant metrics on Lie groups. An existence result for homogeneous geodesics

Isometries of compact Riemannian manifolds. Outline of the Myers-Steenrod theorem about the ìsometry group of a Riemannian manifold. Maximum dimension of the isometry group. Divergence theorem. Killing vector fields on compact manifolds: Bochner's theorem. Every Riemannian homogeneous space with negative semi-definite definite Ricci tensor is a flat torus. Killing vector fields on compact manifolds with positive curvature: Berger's theorem.

Exponential map and Jacobi fields. Exponential map of a Riemannian manifold and its naturality. Normal neighbourhoods and geodesic balls. Jacobi vector fields. Every Killing vector fields restricts to a Jacobi field along a geodesic. Maximal dimension of the Lie algebra of Killing vector fields. Geodetic completeness of Riemannian homogeneous spaces.

Riemannian symmetric spaces.
Geodesic reflections. Riemannian symmetric spaces. Examples. Canonic representation of a symmetric space Riemannian as a homogeneous reductive space; Cartan decomposition. Curvature and Ricci tensor of a symmetric space. Compact and non-compact type spaces and sign of the sectional curvatures. Every Riemannian homogeneous space with nonpositive sectional cuvature and negative definite Ricci tensor is simply connected (theorem of Kobayashi).

Teaching methods: Lectures and exercise

Auxiliary teaching:

Assessment methods: Oral exam

Bibliography:

1) M. Falcitelli, S. Ianus, A. M. Pastore: Riemannian submersions and related topics, World Scientific Publishing Co, Inc., River Edge, NJ, 2004.

2) S. Kobayashi, K. Nomizu: Foundations of differential geometry. Vol. II, John Wiley & Sons, Inc., New York, 1969.
3) J.M. Lee: Riemannian manifolds. Graduate Texts in Mathematics 176, Springer-Verlag, New York, 1997.
4) B. O'Neill: Semi-Riemannian geometry. Academic Press, San Diego, 1983.
5) M. Postnikov: Geometry VI. Riemannian geometry. Encyclopaedia of Mathematical Sciences 91, Springer-Verlag, Berlin, 2001.