Academic subject: Advanced Geometry 1				
Degree Class: LM-40		**Degree Course:** Mathematics	**Academic Year:** 2018/2019	
		Kind of class: Mandatory/optional depending on the curriculum	**Year:** 2	**Period:** 1
			ECTS: 7 divided into **ECTS lessons:** 6.5 **ECTS exe:** 0.5	
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe: 8 in–class study: 60 out–of–class study: 115				
Language: Italian	**Compulsory Attendance:** no			
Subject Teacher: Maria Falcitelli	**Tel:** 39 0805442844 **e–mail:** maria.falcitelli@uniba.it	**Office:** Department of Mathematics Room 9, Floor 3	**Office days and hours:** Thursday: 11-13. Other days, by appointment	
Prerequisites: Mathematical knowledge acquired during the first degree in Mathematics. In particular: linear Algebra, general Topology, classical Mathematical Analysis, affine and projective Geometry, basic concepts occurring in differential Geometry				
Educational objectives: Acquiring new concepts and basic methods occurring in modern Differential Geometry, in particular in Riemannian Geometry.				
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring new concepts and methods of proof. **Applying knowledge and understanding:** The acquired knowledge is useful in various contexts, such as in theoretical Physics. **Making judgements:** Ability in recognizing new techniques used in problem solving. **Communication:** Students should acquire the mathematical formalism which is necessary to analyze advanced problems. **Lifelong learning skills:** Relating the main concepts occurring in various mathematical and Physical disciplines.			

Course program

Fundamental examples of smooth manifolds.
The Euclidean space R^n. The sphere $S^n(r)$. The real projective space $P_n(R)$ and the antipodal map. The hyperbolic space H^n_r.

The tensor algebra of a manifold.
The tensor algebra on a vector space. Tensor fields of type (r,s) on a manifold: definition and properties. The tensor algebra of a manifold. Contractions. Symmetric, skew-symmetric tensors on a vector space. Symmetric tensor fields, differential forms on a manifold. The exterior product and the algebra of differential forms. The exterior differential.

Derivations of the tensor algebra.
Definition and main properties of a derivation of the tensor algebra. Examples: the derivation associated with a (1,1)-tensor field, the Lie derivative with respect to a vector field. A representation theorem of derivations.

Linear connections.
Definition of a linear connection. The covariant derivative of a tensor field with respect to a connection. The canonical connection on R^n. The localizability property and a representation theorem. The covariant derivative of a vector field along a curve. Parallel vector fields, geodesic curves: definition and equations. The parallel transport along a curve. The torsion and the curvature tensors of a connection. Symmetric, flat connections. Bianchi identities.

Riemannian manifolds.

Riemannian metrics on a manifold. The metric induced on a submanifold of a Riemannian manifold. Examples. The scalar product of two tensor fields. The musical isomorphisms. The gradient od a smooth function. The Levi-Civita connection on a Riemannian manifold and the Christoffel symbols. Examples. The parallel transport along a curve induced by the Levi-Civita connection. The distance between two points in a Riemannian manifold. Complete, geodesically complete manifolds. Conformal changes of a metric.

Riemannian curvature.
The Riemannian curvature tensor: definition and properties. Sectional curvatures. Manifolds with pointwise sectional curvature. The Schur lemma. Space-forms: definition and main examples. Riemannian covering spaces. Example: the n-sphere as a Riemannian covering of $P_n(R)$. Complete, connected, simply connected space-forms: a classification theorem. Ricci tensor and scalar curvature. Einstein manifolds. A characterization of Einstein manifolds in dimension 3.

Riemannian submanifolds.
Riemannian submanifolds of a Riemannian manifold: definition and examples. The normal bundle, normal vector fields. The Gauss and Weingarten equations. The second fundamental form, the Weingarten operators: definition and properties. The mean curvature vector. Totally geodesic, totally umbilical, minimal submanifolds. Principal curvatures. Some curvature properties of a submanifold: Gauss, Codazzi, Ricci equations. Hypersurfaces in R^{n+1}.

Teaching methods: Lectures and exercise lessons.

Auxiliary teaching:

Assessment methods:
Oral exam.

Bibliography:
T. Aubin: A course in Differential Geometry, American Mathematical Society
B. Y. Chen: Geometry of submanifolds, Marcel Dekker
W. Klingenberg: Riemannian Geometry, Walter de Gruyter
S. Kobayashi, K. Nomizu: Foundations of Differential Geometry, Vol. I,II, Interscience Publishers
G. Walschap: Metric structures in Differential Geometry, Springer.