Academic subject: Riemannian Geometry				
Degree Class: LM-40 – Matematica		**Degree Course:** Mathematics	**Academic Year:** 2019/2020	
		Kind of class: optional	**Year:**	**Period:** II semester
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115				
Language: Italian	**Compulsory Attendance:** no			
Subject Teacher: Luigia Di Terlizzi **Collaborator:** Dileo Giulia	**Tel:** 080 5442694 **e–mail:** luigia.diterlizzi@uniba.it	**Office:** Department of Mathematics Room 19, Floor III	**Office days and hours:** By appointment on Microsoft Teams	
Prerequisites: Basic knowledge in differential geometry: differentiable manifolds, tangent and cotangent spaces, tangent bundle. Tensorial algebra and tensorial calculus. Elements of Riemannian geometry.				
Educational objectives: Knowledge of the most important results in Riemannian geometry, with special attention to Hermitian and contact geometry.				
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring results in the mostly investigated research fields of Riemannian Geometry, allowing to comprehend advanced textbooks and recent publications. **Applying knowledge and understanding:** Acquiring proof techniques in Hermitian and contact geometry, together with the knowledge of fundamental examples. **Making judgements:** Ability to analyze the consistency of mathematical arguments, under the formal, logical and technical point of view. Students should become able to prove autonomously properties dealing with the program topics. **Communication:** Students should acquire the mathematical language and formalism necessary to the comprehension and exposition of concepts and results concerning the studied theory. **Lifelong learning skills:** Improve learning methods acquired during previous years, through the practice in exposing results and solving problems.			

Course program

Complex vector spaces. Complexification of real vector spaces and real linear maps. Complexification of the dual vector space. Complex structures on real vector spaces. Canonical complex structure on R^{2n}. C-linear maps between complex vector spaces. GL(n,C) as subgroup GL(2n,R). Positively oriented bases in a complex vector space. Complexification of a complex vector space. Vectors of type (1,0) and (0,1). Forms of type (1,0) and (0,1). Decomposition of the complexified exterior algebra.

Almost complex manifolds. Almost complex structures on differentiable manifolds. Adapted frames. Orientability of almost complex manifolds. Canonical almost complex structure on C^n. Vector fields and 1-forms of type (1,0) and (0,1). Decomposition of the complexified exterior bundle. The Nijenhuis tensor associated to an almost complex structure: necessary and sufficient conditions for its vanishing.

Complex manifolds. Holomorphic functions and Cauchy-Riemann equations. Complex manifolds. Holomorphic maps between complex manifolds. Examples: complex projective space, S^2, biholomorphism between S^2 e CP^1, complex Lie groups. Canonical almost complex structure on a complex manifold. Adapted local frames and coframes. The Newlander-Nirenberg Theorem. Differential operators on complex manifolds. Complex structures on Riemannian oriented surfaces. Characterizations of holomorphic functions. Holomorphic vector fields and holomorphic 1-forms. Real holomorphic vector fields.

Almost Hermitian manifolds. Hermitian inner products on complex vector spaces. The standard Hermitian inner product on C^n. Hermitian matrices. Orthonormal bases and unitary matrices. Real part and imaginary part of a Hermitian inner product. 2-fundamental form and orthonormal J-bases. U(n) as a subgroup of SO(2n). Complexification of a Hermitian inner product and its fundamental 2-form. Almost Hermitian manifolds. Existence of Hermitian metrics. C^n as a Hermitian manifold. Adapted local orthonormal frames. Non-degeneracy of the fundamental 2-form. The Levi-Civita connection: covariant derivatives of the almost complex structure and the fundamental 2-form. The Nijenhuis tensor of an almost Hermitian manifold. Some classes of almost Hermitian manifolds. Holomorphic sectional curvature.

Kähler manifolds. Definition and characterization of Kähler. Kähler structure on S^2 and on Riemannian oriented surfaces. Riemannian curvature properties for a Kähler manifold. Kähler manifolds with constant sectional curvature. Kähler manifolds with constant holomorphic sectional curvature. Hermitian metrics in complex coordinates. Characterization of Kähler metrics. Kähler potential. The Bergman metric on the complex disk. The Fubini-Study metric on the complex projective space. Classification of Kähler manifolds of constant holomorphic sectional curvature.

Symplectic manifolds. Existence of canonical basis for an skew-symmetric form. Symplectic forms on a vector space and symplectic basis. Characterization of symplectic forms. Almost symplectic and symplectic manifolds. Almost Hermitian manifolds as almost symplectic manifolds. The standard symplectic structure on $\mathbf{R}^{2n}$. Darboux Theorem for the symplectic manifolds. Existence of an almost complex structure on a symplectic manifold.

Almost contact and contact manifolds. Contact elements on a manifold. Contact structure on a manifold as distribution of contact elements. 1-forms that (locally and globally) define a contact structure. The dimension of a differential manifold equipped with a contact structure is odd. Orientability of a   manifold equipped with a contact structure. Standard contact structure on $\mathbf{R}^{2n+1}$. (φ, ξ, η)-structures. Existence of compatible metrics on a paracompact manifold equipped with a (φ, ξ, η)-structures. Existence of a (φ, ξ, η)-structure on a contact manifold, proved using the polar decomposition Theorem.
Almost contact metric manifolds. Existence of local orthonormal frames adapted to an almost contact metric structure. Contact structures: definition, properties and examples. Rank of an almost contact structure. Almost contact structure on an orientable hypersurface of an almost Hermitian manifold. Example: the structure induced on the sphere S^{2n+1} b the canonical Hermitian structure of $\mathbf{R}^{2n+2}$. Condition for the induced structure on an hypersurface of a Kähler manifold to be contact. Normal almost contact manifolds: definition and properties. The fundamental 2-form of an almost contact metric manifold. Sasakian manifolds: definition, algebraic characterization and examples. The tensor fields $N^{(1)}$ $N^{(2)}$, $N^{(3)}$, $N^{(4)}$ and the normality condition on an almost contact manifold. *D*-deformations.
K-contact manifolds and characterization. Cosymplectic manifolds and characterization. Integrability of the contact structure of a cosymplectic manifold. Analogy between the cosymplectic and the Kähler manifolds. Definition of the nearly cosymplectic and of the almost cosymplectic manifolds and some properties. Sasakian manifolds and caracterizations.

Curvature. Curvature identities for the contact metric manifolds. ξ-sectional curvature of the K-contact manifolds. (κ, μ)-contact manifolds. Properties of the Ricci tensor for the K-contact manifolds. Olszak's Theorem for the contact manifolds with constant sectional cutvature. Contact metric manifolds such that $R_{XY}\xi=0$ and local decomposition theorem. Definition of φ-sections, of φ-sectional curvature, and manifolds with (pointwise) constant φ-sectional curvature. Sasakian space forms. The η-Einstein and the Sasaki-Einstein manifolds.

Quasi-Sasakian manifolds. Quasi-Sasakian manifolds and the Kanemaki's characterization. The indicator tensor field. The case of Sasakian and cosymplectic manifolds. Examples. Local decomposition theorem of the quasi-Sasakian manifolds with parallel indicator field and constant rank. Definition of β-Kenmotsu manifolds, properties and examples. The leaves of a β-Kenmotsu manifold are Kähler manifolds.

Teaching methods:
Lectures and exercise sections

Auxiliary teaching:

Assessment methods:
Oral exam

Bibliography:

D. E. Blair, Riemannian geometry of contact and symplectic manifolds. Second edition. Progress in Mthematics, 203. Birkäuser, Boston, 2010.

S. Kobayashi, K. Nomizu, Foundations of differential geometry. Interscience Publishers.

A. Moroianu, Lectures on Kähler geometry. London Mathematical Society Student Texts, 69. Cambridge University Press, Cambridge, 2007.

Ana Cannas da Silva, Lectures on Symplectic Geometry. Springer