Academic subject: Nonlinear Analysis

Degree Class: L-35 Scienze Matematiche	**Degree Course:** Mathematics	**Academic Year:** 2018/2019

	Kind of class: Optional	**Year:**	**Period:** 2

			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5

Time management, hours, in–class study hours, out–of–class study hours
lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115

Language: Italian	**Compulsory Attendance:** no	

Subject Teacher: Anna Maria Candela Silvia Cingolani	**Tel:** +39-0805442669 **e–mail:** annamaria.candela@uniba.it	**Office:** Department of Mathematics Floor II, Room 20	**Office days and hours:** Days and times have to be arranged by email

Prerequisites:
In addition to the mathematical knowledge which usually is acquired during the first two years of a degree of L–35 class, they are required language and techniques of modern analysis such as basic theory of Hilbert spaces and L^p spaces.

Educational objectives:
Acquiring instruments of Functional Analysis which allow one to study some differential equations as variational problems, i.e. defining a suitable functional on a Hilbert space looking for its minimum, or, more in general, for its critical points. Proving some abstract existence theorems which are applied for searching solutions of some linear and nonlinear differential equations coming from Mathematical Physics.

Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring fundamental concepts of variational methods, their related proof techniques and how to apply them for studying some differential equations. **Applying knowledge and understanding:** The acquired variational methods apply for studying some nonlinear differential equations which describe some classical problems in Geometry and in Mathematical Physics. **Making judgements:** Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the correct methods required for studying nonlinear differential equations with variational structure. **Communication:** Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve variational problems. **Lifelong learning skills:** Acquiring suitable learning methods, supported by text consultation and by solving some nonlinear model differential equations.

Course program
Function spaces Background knowledge on spaces of C^k functions and on Lebesgue spaces. Elements of distribution theory, Sobolev spaces and their main properties.
Linear problems Elements of spectral theory. Symmetric and self-adjoint operators. Friedrichs extension. Self-adjoint extension and its spectral properties for the Laplace operator with omogeneous Dirichlet boundary conditions. Weak solutions of elliptic boundary value problems. Regularity theorems.
Differential calculus in Banach spaces Fréchet and Gâteaux derivatives. Theorem of Total Differential. Properties and examples of differentiable functionals. Higher order derivatives. Critical points and local extrema. Weak convergence and Weierstrass Theorem in Banach spaces.
Nonlinear differential problems Nemytskii operators on Sobolev spaces. Weak solutions and variational principles for some nonlinear differential problems. Hamilton's principle of least action. Weierstrass Theorem and existence of weak solutions. Ekeland’s variational principle.
Examples Critical points of functionals on manifolds and some problems with constraints: non-homogeneous elliptic problems, dynamical systems on manifolds and their trajectories joining two fixed points, nonlinear eigenvalue

problems.
Unbounded functionals Variational problems with unbounded functionals. The Palais-Smale condition and its variants. Deformation Lemma. Mountain Pass Theorem. Three Solutions Theorem. Applications to some nonlinear differential equations.

Teaching methods:
Lectures and exercise sessions

Auxiliary teaching:

Assessment methods:
Oral exam

Bibliography:

- R.A. Adams & J.J.F. Fournier, "Sobolev Spaces" (2nd Ed.), Academic Press, Amsterdam, 2003
- A. Ambrosetti & G. Prodi, "A Primer of Nonlinear Analysis", Cambridge University Press, Cambridge, 1993
- H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations", Springer, New York, 2011
- D. Costa, "An Invitation to Variational Methods in Differential Equations", Birkhäuser, Basel, 2007
- M. Struwe, "Variational Methods. Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems" (4th Ed.), Ergeb. Math. Grenzgeb. (4) **34**, Springer–Verlag, Berlin, 2008