Academic subject: Riemannian Geometry				
Degree Class: LM-40 – Matematica		**Degree Course:** Mathematics	**Academic Year:** 2018/2019	
		Kind of class: optional	**Year:**	**Period:** II semester
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115				
Language: Italian	**Compulsory Attendance:** no			
Subject Teacher: Giulia Dileo	**Tel:** **e–mail:** giulia.dileo@uniba.it	**Office:** Department of Mathematics Room 35, Floor II	**Office days and hours:** By appointment to be agreed by e-mail.	

Prerequisites: Basic knowledge in differential geometry: differentiable manifolds, tangent and cotangent spaces, tangent bundle. Tensorial algebra and tensorial calculus. Elements of Riemannian geometry.

Educational objectives: Knowledge of the most important results in Riemannian geometry, with special attention to Hermitian and contact geometry.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
Acquiring results in the mostly investigated research fields of Riemannian Geometry, allowing to comprehend advanced textbooks and recent publications.

Applying knowledge and understanding:
Acquiring proof techniques in Hermitian and contact geometry, together with the knowledge of fundamental examples.

Making judgements:
Ability to analyze the consistency of mathematical arguments, under the formal, logical and technical point of view. Students should become able to prove autonomously properties dealing with the program topics.

Communication:
Students should acquire the mathematical language and formalism necessary to the comprehension and exposition of concepts and results concerning the studied theory.

Lifelong learning skills:
Improve learning methods acquired during previous years, through the practice in exposing results and solving problems.

Course program

Complex vector spaces. Complexification of real vector spaces and real linear maps. Complexification of the dual vector space. Complex structures on real vector spaces. Canonical complex structure on R^{2n}. C-linear maps between complex vector spaces. GL(n,C) as subgroup GL(2n,R). Positively oriented bases in a complex vector space. Complexification of a complex vector space. Vectors of type (1,0) and (0,1). Forms of type (1,0) and (0,1). Decomposition of the complexified exterior algebra.

Almost complex manifolds. Almost complex structures on differentiable manifolds. Adapted frames. Orientability of almost complex manifolds. Canonical almost complex structure on C^n. Vector fields and 1-forms of type (1,0) and (0,1). Decomposition of the complexified exterior bundle. The Nijenhuis tensor associated to an almost complex structure: necessary and sufficient conditions for its vanishing.

Complex manifolds. Holomorphic functions and Cauchy-Riemann equations. Complex manifolds. Holomorphic maps between complex manifolds. Examples: complex projective space, S^2, biholomorphism between S^2 e CP^1, complex Lie groups. Canonical almost complex structure on a complex manifold. Adapted local frames and coframes. The Newlander-Nirenberg Theorem. Differential operators on complex manifolds. Complex structures on Riemannian oriented surfaces. Characterizations of holomorphic functions. Holomorphic vector fields and holomorphic 1-forms. Real holomorphic vector fields.

Almost Hermitian manifolds. Hermitian inner products on complex vector spaces. The standard Hermitian inner product on C^n. Orthonormal bases and unitary matrices. Real part and imaginary part of a Hermitian inner product. 2-fundamental form and orthonormal J-bases. U(n) as a subgroup of SO(2n). Complexification of a Hermitian inner product and its fundamental 2-form. Almost Hermitian manifolds. Existence of Hermitian metrics. C^n as a Hermitian manifold. Adapted local orthonormal frames. Non-degeneracy of the fundamental 2-form. The Levi-Civita connection: covariant derivatives of the almost complex structure and the fundamental 2-form. The Nijenhuis tensor of an almost Hermitian manifold. Some classes of almost Hermitian manifolds. Holomorphic sectional curvature.

Kähler manifolds. Definition and characterization of Kähler. Kähler structure on S^2 and on Riemannian oriented surfaces. Riemannian curvature properties for a Kähler manifold. Kähler manifolds with constant sectional curvature. Kähler manifolds with constant holomorphic sectional curvature. Hermitian metrics in complex coordinates. Characterization of Kähler metrics. Kähler potential. The Bergman metric on the complex disk. The Fubini-Study metric on the complex projective space. Classification of Kähler manifolds of constant holomorphic sectional curvature.

Simplectic manifolds. Existence of canonical bases for a bilinear skew-symmetric form. Symplectic forms on vector spaces and symplectic vector spaces. Almost symplectic and symplectic manifolds. Almost Hermitian manifolds as symplectic manifolds. Standard symplectic structure on R^{2n}. The Darboux theorem for the symplectic manifolds. Theorem of existence of an almost complex metric structure on a symplectic manifold.

Almost contact and contact manifolds. Contact elements on a manifold. Contact structure on a manifold as a distribution on contact elements. Contact structure locally and globally defined by a 1-form. The dimension of a manifold with a contact structure is odd. A manifold equipped with a contact structure is orientable. The canonical contact structure on R^{2n+1}. (φ, ξ, η)-structures on a manifold and existence of compatible metrics when the manifold is paracompact. Proof of the existence of a (φ, ξ, η)-structure on a contact manifold using the polar decomposition theorem. Normal contact manifolds. The tensor fields N^1, N^2, N^3,N^4. Rank of an almost contact structure. Existence of an almost contact structure on an orientable hypersurface of an almost Hermitian manifold. The almost complex structure on the product of two almost contact manifolds introduced by Morimoto. Compatible metrics. The Reeb vector field is parallel with respect to the Levi-Civita connection of a contact metric manifold. The tensor field h on a contact manifold and its properties. K-contact manifolds. Cosymplectic manifolds and properties. Sasakian manifolds and the Sasakian structure on an odd-dimensional sphere.

Some questions regarding the curvature of the contact metric manifolds. Some curvature identities on a contact metric manifold. The ξ-sectional curvature on the K-contact manifolds. A characterization of the Sasakian manifolds by mean of the curvature. The Ricci tensor field for K-contact manifolds. Contact metric manifolds of constant sectional curvature and the Theorem by Olszak. Contact metric manifolds such that $R_{XY}\xi=0$ and the local decomposition theorem. The contact (κ,μ)-manifolds: the eigenvalues of the tensor field h and the eigenspace distributions. φ-sectional curvatures of Sasakian manifolds. The φ-sectional curvatures determine the curvature of a Sasakian manifold. If the φ-sectional curvature does not depend on the φ-section it does not depend on the point. Sasakian space forms. The three classes of Sasakian space form.

Quasi-Sasakian manifolds. Characterization of quasi-Sasakian manifolds by Kanemaki. Indicator tensor field of a quasi-Sasakian structure. Characterization of the Sasakian and of the cosymplectic manifolds by the indicator tensor field and examples.

Teaching methods: Lectures and exercise sections
Auxiliary teaching:
Assessment methods: Oral exam
Bibliography: D. E. Blair, Riemannian geometry of contact and symplectic manifolds. Second edition. Progress in Mthematics, 203. Birkäuser, Boston, 2010. S. Kobayashi, K. Nomizu, Foundations of differential geometry. Interscience Publishers. A. Moroianu, Lectures on Kähler geometry. London Mathematical Society Student Texts, 69. Cambridge University Press, Cambridge, 2007.