Academic subject: Numerical Calculus 2				
Degree Class: L-35 – Scienze Matematiche		**Degree Course:** Mathematics	**Academic Year:** 2017/2018	
		Kind of class: Optional	**Year:** 3	**Period:** 2
			ECTS: 7 divided into **ECTS lessons: 5** **ECTS exe/lab/tutor: 2**	

Time management, hours, in–class study hours, out–of–class study hours

lesson: 40 exe/lab/tutor: 30 in–class study: 70 out–of–class study: 105

Language: Italian	**Compulsory Attendance:** no		
Subject Teachers: Felice Iavernaro	**Tel:** +39 080 5442703 **e–mail:** felice.iavernaro@uniba.it	**Office:** Department of Mathematics Room 2 , 4thFloor	**Office days and hours:** Monday 14:00—16:00. Other days by appointment only.
Roberto Garrappa	**Tel:** +39 080 54422685 **e-mail:** roberto.garrappa@uniba.it	Department of Mathematics Room 7 , 3thFloor	Thursday 14:00—16:00. Other days by appointment only.

Prerequisites:
The knowledge gained in the course “Numerical Claculus 1”, programming in Matlab, classical analysis of one and several variables, fundamental linear algebra.

Educational objectives:
Acquiring numerical methods and programming techniques in the context of interpolation, data fitting, numerical integration, numerical solution of ordinary differential equations.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
- Acquiring a knowledge of the most important numerical methods able to solve mathematical real-world problems, with particular reference to data fitting, numerical integration and initial value problems.
- Understanding and being able to explain issues related to the use of a computer for solving the above mentioned mathematical problems.

Applying knowledge and understanding:
- Capability of solving mathematical problems by means of suitable algorithms enjoying good stability properties and low cost implementation.
- Acquiring skills in programming, testing numerical algorithms and consistently interpreting computer results.

Making judgements:
- Being able to detect a proper numerical method to solve a given mathematical problem among those analyzed during the lectures.

Communication:
- Being able to provide rigorous definitions of the analyzed mathematical problems and to discuss the related numerical methods, outlining their most important features.

Lifelong learning skills:
- Capability of studying and solving problems similar, but not necessarily equivalent, to those faced during the teaching activities.

Course program

1. **INTERPOLATION.** The polynomial interpolation problem: existence and uniqueness theorem. Undetermined coefficient method: computational cost and stability issues. Lagrange formula and barycentric formulations. Remainder term in the polynomial interpolation. Convergence properties. Stability properties: Lebesgue function and Lebesgue constant. Newton divided differences interpolation formula: Newton basis and divided differences.

Newton-Gregory forward difference formula. Runge's phenomenon. Chebyshev nodes and their optimality property. Chebychev-Lobatto nodes and related barycentric formula. Convergence results for differentiable and analytic functions. Hermite interpolation: extensions of Lagrange and Newton interpolation formulae, remainder theorem and computational properties. Linear splines: construction of a basis, stability, remainder and convergence properties. Cubic spline interpolation: extra boundary conditions, construction and implementation; hints about convergence and minimal curvature property. Generalized polynomials and Haar condition. Trigonometric interpolation: DFT and IDFT algorithms, computational complexity and implementation; some applications to the signal processing.

2. **APPROXIMATION.** Overdetermined linear systems. Least squares approximation: problem definition, existence and uniqueness theorem. Least Squares fitting-polynomial. Linear regression line. Coefficient of determination. QR factorization of a rectangular matrix by means of Householder elementary transformations. Use of QR factorization for solving least squares problems. Singular Value Decomposition: existence result and fundamental properties. Solution of the least squares problem through the SVD decomposition. Pseudoinverse and its properties. Conditioning of the least squares problem. Bidiagonalization of a matrix by means of Householder elementary transformations. Algorithm for the SVD computation by means of Givens rotations. Some applications of the SVD: computation of the rank and the 2-norm of a matrix; best low-rank approximation of a matrix; regularizing an ill-conditioned linear system; digital image compression; latent semantic analysis; principal component analysis. Least squares problem in L2([a,b]): problem definition, use of an orthogonal basis, Bessel's inequality and Parseval's identity. Error estimation and implementation aspects. Orthogonal polynomials: Gram-Schmidt ortogonalization process, three-term recurrence relation, property of the roots. Rodrigues' formula and study of the family of Legendre polynomials, first and second kind Chebyshev polynomials, Laguerre and Hermite polynomials. Truncated Fourier series. Padé rational approximation.

3. **NUMERICAL INTEGRATION.** Interpolation quadrature formulae, degree of precision and error analysis. Rectangle, trapezoidal and Simpson rules: definition and error analysis. Newton-Cotés formulae. Conditioning of the problem and stability of an interpolation quadrature formula. Composite trapezoidal and Simpson's rules, error analysis. Recursive algorithm with automatic step control based on the trapezoidal Simpson's rules. Higher-order formulae, weights positivity and convergence properties. Gauss-Legendre, Radau and Lobatto formulae. Hints on adaptive integration techniques.

4. **NUMERICAL METHODS FOR INITIAL-VALUE PROBLEMS.** One-step methods: definition, consistency and convergence analysis. Explicit and implicit Euler methods, implicit midpoint and trapezoidal methods. Collocation methods: definition and link to Runge-Kutta methods. Existence and uniqueness of the collocation polynomial. Order analysis. Gauss, Radau, Lobatto collocation methods.

5. **PROGRAMMING ENVIRONMENT FOR SCIENTIFIC COMPUTING.** The implementation of the algorithms introduced in the course will be carried out in Matlab, which is a scientific computing environment equipped with a number of built-in functions and programming instructions. A particular attention will be paid to the study of the behavior of the solutions of a given problem in finite arithmetic.

Teaching methods:
Lectures and exercise sessions. Exercise sessions in the Computer Centre,

Auxiliary teaching:
Handouts, notes and Matlab codes will be made available through an e-learning platform. Log-in information will be provided at the course starting days.

Assessment methods:
The exam consists of an oral test which includes a discussion of the Matlab codes prepared by the student.

Bibliography:

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- G.H. Golub, C.F. Van Loan, Matrix computations. Fourth edition. Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, Baltimore, MD, 2013.
- R. Bevilacqua, D. Bini, M. Capovani, O. Menchi, Metodi Numerici, Zanichelli, Bologna, 1992.

- E. Hairer, S.P. Nørsett, G. Wanner, Solving Ordinary Differential Equations I. Nonstiff Problems. Springer Series in Comput. Mathematics, Vol. 8, Springer- Verlag 1987, Second revised edition 1993.
- D. Bini, M. Capovani, O. Menchi, Metodi numerici per l'algebra lineare, Zanichelli.
- K.E. Atkinson, An introduction to numerical analysis, Wiley, 1989
- D.M. Young, R.T. Gregory, A survey of numerical mathematics, Vol. I, Dover, 1988