Academic subject: Mathematical Analysis no. 4

Degree Class: L-35 (Scienze Matematiche)	**Degree Course:** Mathematics	**Academic Year:** 2017/2018
	Kind of class: mandatory	**Year:** 2 **Period:** 2
		ECTS: 8 divided into **ECTS lessons:** 5 **ECTS exe/lab/tutor:** 3

Time management, hours, in–class study hours, out–of–class study hours

lesson: 40 exe/lab/tutor: 30 in–class study: 70 out–of–class study: 130

Language: Italian	**Compulsory Attendance:** no	

Subject Teacher: Francesco Altomare	**Tel:** +39 080 5442672 **e–mail:** francesco.altomare@uniba.it	**Office:** Department of Mathematics Room 6, Floor 3	**Office days and hours:** Monday, 10:00 – 12:00 Tuesday, 10:30 – 12:30

Prerequisites:
Mathematical knowledge which usually is acquired during the first year and the first semester of the second year of a degree of L–35 class. Especially:
classical mathematical analysis of one and several variable, differential calculus, integral calculus, elements of topology in metric spaces and Euclidean spaces, linear algebra.

Educational objectives:
To acquire further knowledge and techniques of classical mathematical analysis, especially elements of the theory of differential equations, Riemann integration theory for functions of several variables, the Peano-Jordan measure on R^n.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
To acquire fundamental concepts in the classical mathematical analysis. To acquire basic mathematical proof techniques.

Applying knowledge and understanding:
The acquired theoretical knowledge is useful in great part of mathematics and its applications.

Making judgements:
Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge.

Communication:
Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems.

Lifelong learning skills:
To acquire suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested throughout the course.

Course program

1. <u>COMPLETE METRIC SPACES AND BANACH SPACES</u>

Cauchy sequences in metric spaces and in normed spaces. Complete metric spaces. Compactness and completeness in metric spaces. Banach spaces. Completeness of the spaces IR^n, $B(X, E)$ and $C_b(X, E)$ (E Banach space). The Banach fixed point theorem.

2. ORDINARY DIFFERENTIAL EQUATIONS AND SYSTEMS

Preliminaries on the integration theory for functions of one variable into IR^n. The fundamental theorem of calculus.

Generalities on first-order ordinary differential equations in normal form and the relevant Cauchy problems. The Cauchy-Lipschitz theorem about existence and uniqueness of solutions of Cauchy problems. The Cauchy-Lipschitz theorem about existence and uniqueness of local solutions of Cauchy problems.

Systems of first-order ordinary differential equations in normal form and the relevant Cauchy problems. Existence and uniqueness of global or local solutions.

Higher order ordinary differential equations in normal form and the relevant Cauchy problems. Existence and uniqueness of global or local solutions.

Complete integrals and singular integrals for ordinary differential equations.

Generalities on ordinary linear differential equations in normal forms. Homogeneous ordinary linear differential equations. Independent integrals and their Wronskian. Complete integrals for homogeneous linear ordinary differential equations. Nonhomogeneous linear ordinary differential equations. Complete integrals for nonhomogeneous linear ordinary differential equations. The Lagrange method of variation of constants. Linear ordinary differential equations with constant coefficients.

Methods of integration of some first-order differential equations in normal form. Separable differential equations, homogeneous differential equations, Bernoulli differential equations. Exact differential equations. Integrating factors of differential equations. Methods of integration of some second-order differential equations. Qualitative analysis of differential equations

3. INTEGRATION OF FUNCTIONS OF SEVERAL VARIABLES AND THE PEANO-JORDAN MEASURE IN IR^n

Intervals of IR^n and their elementary content. Partitions of intervals of IR^n and the relevant cell subdivisions. Simple functions on intervals and their integrals. Riemann integrable functions on intervals and their integral. Riemann integrals and Darboux lower and upper sums. Linearity, additivity, positivity and monotonicity of the Riemann integral. Convergence theorem under the integral sign. Regulated functions on intervals and their integrability. Integrability of continuous functions on intervals.

Supports of real-valued functions on IR^n. Definition of Riemann integrability for bounded functions on IR^n with compact support and their Riemann integral. Riemann negligible subsets of IR^n. Noteworthy examples and properties. Riemann integrability of bounded functions on IR^n with compact support which are continuous except for a negligible set.

Definition of Riemann integrability for bounded functions defined on a bounded subset of IR^n and their integral. Bounded subsets with negligible boundary and Riemann integrability of bounded functions on them which are continuous except for a negligible set. Linearity and additivity of the Riemann integral. Convergence theorem under the integral sign.

Figures of IR^n and their premeasures. Inner and outer Peano-Jordan measures of bounded subsets of IR^n. Peano-Jordan measurable subsets of IR^n and their Peano-Jordan measure. Peano-Jordan measurable subsets and Riemann integrability of constant functions. Peano-Jordan measure as integral of the characteristic function. Peano-Jordan measurable subsets and subsets with negligible boundary. Peano-Jordan measurability of the subgraph of integrable positive functions and their measure.

The calculus of integrals. Double integrals over normal domains of IR^2 and areas of two-dimensional domains. Change of variables in multiple integrals. Polar coordinates. Triple integrals over normal domains of IR^3 and volumes of three-dimensional domains.

3. CURVES, LINE INTEGRALS AND DIFFERENTIAL FORMS

Curves in IR^n. Regular curves. Length of a curve. Oriented curves. Parametrization by arc length of curves. Line integral of functions along a curve. Differential forms. Line integral of differential forms along a curve. Exact differential forms. Closed differential forms. The Gauss-Green theorem.
Teaching methods: Lectures and exercise sessions.
Auxiliary teaching:
Assessment methods: Written and oral exam
Bibliography: **<u>REFERENCE TEXTBOOKS</u>** [1] P. MARCELLINI – C. SBORDONE, Elementi di Analisi Matematica due, Liguori Editore, Napoli, 2001. [2] P. MARCELLINI – C. SBORDONE, Esercitazioni di Matematica, 2° Volume, Parte I e Parte II, Liguori Editore, Napoli, 1989. **<u>FURTHER SUGGESTED READINGS</u>** [1] N. FUSCO - P. MARCELLINI – C. SBORDONE, Analisi Matematica due, Liguori Editore, Napoli, 1996. [2] J. LELONG-FERRAND – J. M. ARNAUDIÈS, Cours de mathématiques, Tome 4, 2e édition, Dunod Université, Paris, 1977. [3] C. D. PAGANI – S. SALSA, Analisi Matematica 2, Seconda edizione, , Zanichelli Editore, Bologna, 2016.