Academic subject: Elements of Advanced Analysis 2

Degree Class: LM–40 – Matematica	**Degree Course:** Mathematics	**Academic Year:** 2017/2018	
	Kind of class: mandatory	**Year:** 1	**Period:** 2

			ECTS: 7 divided into **ECTS lessons:** 6 **ECTS exe/lab:** 1

Time management, hours, in–class study hours, out–of–class study hours

lesson: 48 exe/lab.: 24 in–class study: 72 out–of–class study: 103

Language:	**Compulsory Attendance:**	
Italian	no	

Subject Teacher:	**Tel:** +39 080 5442655	**Office:**	**Office days and hours:**
Enrico Jannelli Cooperates: Marcello D'Abbicco	**e–mail:** enrico.jannelli@uniba.it	Department of Mathematics Room 6, II Floor	Tuesday 11–13. Other days and times by appointment.

Prerequisites:
Mathematical knowledge which usually is acquired during a degree of L–35 class. Especially: classical analysis of one and several variables, general topology, linear algebra, Lebesgue measure and integration theory.

Educational objectives:
Acquiring language and techniques of modern analysis, especially Fourier transform, Banach spaces, weak convergence, distribution theory, Sobolev spaces.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
Acquiring fundamental concepts in advanced modern real and functional analysis. Acquiring basic mathematical proof techniques.

Applying knowledge and understanding:
The acquired theoretical knowledge is useful in large part of mathematics and its applications.

Making judgements:
Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge.

Communication:
Students should acquire the mathematical language and formalism that are necessary to read and comprehend textbooks, to expound the acquired knowledge, and to describe, analyze and solve problems.

Lifelong learning skills:
Acquiring suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested during the course.

Course program

Real Analysis

1. Measure in product spaces: Halmos and Kahn-Kolmogorov abstract theorems – product measure – Fubini-Tonelli theorem – convolution product – Young's theorem – support and regularity of convolutions – Dirac sequences – L^p, pointwise and uniform convergence of convolution with Dirac sequences – Dirac delta as the unity of the convolution product – the fundamental lemma of the calculus of variations.

2. Fourier transform : definition and elementary properties of Fourier transform – the inversion theorem in L^1 – the Fourier transform of some relevant kernels – Fourier transform and derivative – Fourier transform and ODEs – the S space – the Fourier transform in the S space – the Fourier transform in L^2: Plancherel's theorem – Riesz-Thorin theorem (only statement) – the Fourier transform in L^p – Laplace equation in the half-plane – heat equation –

Schrödinger equation – wave equation.

Functional Analysis

3. Elementary theory of Banach spaces: definition, equivalence between continuity and boundedness for linear operators – Baire's theorem – Banach–Steinhaus theorem – open mapping theorem – some properties of Fourier spaces in non hilbertian spaces – Hahn–Banach theorem.

4. Weak convergence (I): dual space of a normed space – bidual space – reflexive spaces – separability and duality – weak and weak–* convergence – elementary properties of weak limits – weakly bounded sets – compactness theorems for weak–* and weak convergence.

5. Weak convergence (II): weak semicontinuity of the norm – uniformly convex spaces (only essentials) – weak convergence and convexity – weak semicontinuity of convex functionals – a minimum theorem for convex functionals – continuous and compact embeddings for spaces $H^s(T)$ e $H^s(T^N)$.

Distributions and Sobolev spaces

6. Introduction to distribution theory: the space $D(\Omega)$ – definition and elementary properties of the distributions, order of a distribution – distributions induced by L^1_{loc} functions – operations with distributions: sum, derivative, multiplication by test functions – support of a distribution – the space $E(\Omega)$ – order of a distribution, any distribution has finite order locally – distributions with compact support – convolution between functions and distributions – convolution between distributions – fundamental solution for PDEs with constant coefficients – fundamental solution for the operator $-\Delta$ – the space *S'* of tempered distributions – slow-growing functions – Fourier transform for tempered distributions – explicit calculation of the Fourier transform of certain tempered distributions – Fourier transform of spherical symmetric functions.

7. Sobolev spaces: definition of $W^{m,p}(\Omega)$ and $H^m(\Omega)$ – completeness of Sobolev spaces – definition of $W_0^{m,p}(\Omega)$ and $H_0^m(\Omega)$ – Theorem: $W^{m,p}(R^N)=W_0^{m,p}(R^N)$ – definition of the spaces $H^s(R^N)$, $s>0$ – embedding theorems of $H^s(R^N)$ into $C^k(R^N)$ – Poincaré inequality – Sobolev spaces on intervals of the real line: continuous embedding of $W^{1,p}(I)$ into $L^\infty(I)$ – Ascoli-Arzelà theorem – compact embedding of $W^{1,p}(I)$ into $C(I)$ – embedding theorems for the spaces $W^{m,p}$ (only statements) – extension operators (only essentials) – Rellich theorems for $W^{m,p}$ (only statements) – critical exponents – representing the dual space of $W_0^{m,p}(\Omega)$ by $W^{-m,p'}(\Omega)$ – some examples of variational problems in Sobolev spaces: problems for the operators $-\Delta$ e $-\Delta+I$ with Dirichlet and Neumann conditions – a nonlinear problem – laplacian eigenvalues – Pohozaev identity.

Teaching methods:
Lectures and exercise sessions.

Auxiliary teaching:
Didactic material available at

http://www.dm.uniba.it/~jannelli/didattica/analisi3/analisi3.htm

Assessment methods:
Oral exam.

Bibliography:

W. RUDIN, *Analisi reale e complessa,* Ed. Boringhieri
H. BREZIS, *Analisi funzionale,* Ed. Liguori
G. GILARDI , *Analisi 3,* Ed. Mc Graw-Hill
S. KESAVAN , *Functional Analysis and Applications,* Ed. J. Wiley & Sons
S. SALSA , *Equazioni a derivate parziali,* Ed. Springer–Verlag Italia

Other didactic material (see above).