<table>
<tr><td colspan="5">Academic subject: Advanced Geometry 1</td></tr>
<tr><td colspan="2">Degree Class: LM-40</td><td colspan="1">Degree Course:
Mathematics</td><td colspan="2">Academic Year:
2017/2018</td></tr>
<tr><td colspan="2"></td><td>Kind of class:
Mandatory/optional depending on the curriculum</td><td>Year:
2</td><td>Period:
1</td></tr>
<tr><td></td><td></td><td></td><td colspan="2">ECTS: 7
divided into
ECTS lessons: 6.5
ECTS exe: 0.5</td></tr>
<tr><td colspan="5">Time management, hours, in–class study hours, out–of–class study hours
lesson: 52 exe: 8 in–class study: 60 out–of–class study: 115</td></tr>
<tr><td>Language:
Italian</td><td>Compulsory Attendance:
no</td><td colspan="3"></td></tr>
<tr><td>Subject Teacher:
Maria Falcitelli</td><td>Tel: 39 0805442844
e–mail:
maria.falcitelli@uniba.it</td><td>Office:
Department of Mathematics
Room 9, Floor 3</td><td colspan="2">Office days and hours:
Thursday: 11-13. Other days, by appointment</td></tr>
<tr><td colspan="5">Prerequisites: Mathematical knowledge acquired during the first degree in Mathematics. In particular: linear Algebra, general Topology, classical Mathematical Analysis, affine and projective Geometry, basic concepts occurring in differential Geometry</td></tr>
<tr><td colspan="5">Educational objectives: Acquiring new concepts and basic methods occurring in modern Differential Geometry, in particular in Riemannian Geometry.</td></tr>
<tr><td>Expected learning outcomes (according to Dublin Descriptors)</td><td colspan="4">Knowledge and understanding: Acquiring new concepts and methods of proof.
Applying knowledge and understanding: The acquired knowledge is useful in various contexts, such as in theoretical Physics.
Making judgements: Ability in recognizing new techniques used in problem solving.
Communication: Students should acquire the mathematical formalism which is necessary to analyze advanced problems.
Lifelong learning skills: Relating the main concepts occurring in various mathematical and Physical disciplines.</td></tr>
</table>

Course program

Fundamental examples of smooth manifolds.
The Euclidean space R^n. The sphere $S^n(r)$. The real projective space $P_n(R)$ and the antipodal map. The hyperbolic space $H^n{}_r$.

The tensor algebra of a manifold.
The tensor algebra on a vector space. Tensor fields of type (r,s) on a manifold: definition and properties. The tensor algebra of a manifold. Contractions. Symmetric, skew-symmetric tensors on a vector space. Symmetric tensor fields, differential forms on a manifold. The exterior product and the algebra of differential forms. The exterior differential.

Derivations of the tensor algebra.
Definition and main properties of a derivation of the tensor algebra. Examples: the derivation associated with a (1,1)-tensor field, the Lie derivative with respect to a vector field. A representation theorem of derivations.

Linear connections.
Definition of a linear connection. The covariant derivative of a tensor field with respect to a connection. The canonical connection on R^n. The localizability property and a representation theorem. The covariant derivative of a vector field along a curve. Parallel vector fields, geodesic curves: definition and equations. The parallel transport along a curve. The torsion and the curvature tensors of a connection. Symmetric, flat connections. Bianchi identities.

Riemannian manifolds.

Riemannian metrics on a manifold. The metric induced on a submanifold of a Riemannian manifold. Examples. The scalar product of two tensor fields. The musical isomorphisms. The gradient od a smooth function. The Levi-Civita connection on a Riemannian manifold and the Christoffel symbols. Examples. The parallel transport along a curve induced by the Levi-Civita connection. The distance between two points in a Riemannian manifold. Complete, geodesically complete manifolds. Conformal changes of a metric.

Riemannian curvature.
The Riemannian curvature tensor: definition and properties. Sectional curvatures. Manifolds with pointwise sectional curvature. The Schur lemma. Space-forms: definition and main examples. Riemannian covering spaces. Example: the n-sphere as a Riemannian covering of $P_n(R)$. Complete, connected, simply connected space-forms: a classification theorem. Ricci tensor and scalar curvature. Einstein manifolds. A characterization of Einstein manifolds in dimension 3.

Riemannian submanifolds.
Riemannian submanifolds of a Riemannian manifold: definition and examples. The normal bundle, normal vector fields. The Gauss and Weingarten equations. The second fundamental form, the Weingarten operators: definition and properties. The mean curvature vector. Totally geodesic, totally umbilical, minimal submanifolds. Principal curvatures. Some curvature properties of a submanifold: Gauss, Codazzi, Ricci equations. Hypersurfaces in R^{n+1}.

Teaching methods: Lectures and exercise lessons.

Auxiliary teaching:

Assessment methods:
Oral exam.

Bibliography:
T. Aubin: A course in Differential Geometry, American Mathematical Society
B. Y. Chen: Geometry of submanifolds, Marcel Dekker
W. Klingenberg: Riemannian Geometry, Walter de Gruyter
S. Kobayashi, K. Nomizu: Foundations of Differential Geometry, Vol. I,II, Interscience Publishers
G. Walschap: Metric structures in Differential Geometry, Springer.