Academic subject: Advanced course in Mathematical Analysis 1				
Degree Class: LM - 40 - Matematica		**Degree Course:** Mathematics	**Academic Year:** 2017/2018	
		Kind of class: mandatory	**Year:** 2	**Period:** 1
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115				
Language: Italian	**Compulsory Attendance:** no			
Subject Teacher: Francesco Altomare	**Tel:** +39 080 5442672 **e–mail:** francesco.altomare@uniba.it	**Office:** Department of Mathematics Room 6 , Floor 3°	**Office days and hours:** Lunedì, 10:00 – 12:00 Martedì, 10:30 – 12:30	
Prerequisites: Mathematical knowledge which usually is acquired with a degree of L–35 class. Especially: classical analysis of one and several variables, metric spaces and Banach spaces, elements of general topology, abstract measure theory and integration.				
Educational objectives: To acquire knowledge and techniques of modern mathematical analysis, especially compact and locally compact topological spaces and continuous function spaces on them, compactness criteria in continuous function spaces, density theorem in continuous function spaces, Borel measures and Radon measures on locally compact spaces, constructive approximation of continuous functions by positive linear operators.				
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** To acquire fundamental concepts and tools in modern mathematical analysis. To acquire basic mathematical proof techniques. **Applying knowledge and understanding:** The acquired theoretical knowledge is useful in great part of mathematics and its applications. **Making judgements:** Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. **Communication:** Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems. **Lifelong learning skills:** To acquire suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested throughout the course.			

Course program

1. **COMPACT AND LOCALLY COMPACT TOPOLOGICAL SPACES [4, 7]**

Compact topological spaces. Locally compact topological spaces. Alexandrov compactification. Urysohn theorem. The theorem on the finite partition of unity. Locally compact spaces countable at infinity. Locally compact spaces with countable bases. Separable topological spaces. Polish topological spaces.

2. CONTINUOUS FUNCTION SPACES [5]

The Banach space C(X) of all (real-valued) continuous functions on a compact space X and its separability. The linear space of all continuous functions with compact support on a locally compact Hausdorff space. Continuous functions converging at infinity on a locally compact space. The Banach spaces $C_0(X)$ and $C_*(X)$ of all continuous functions vanishing at infinity, resp. converging at infinity, on a locally compact space X. Separability of the spaces $C_0(X)$ and $C_*(X)$.

3. COMPACTNESS THEOREMS IN SPACES OF CONTINUOUS FUNCTIONS [5]

Equicontinuous subsets of mappings. Equicontinuous subsets of linear mappings. Uniformly convergent sequences of mappings and equicontinuity. A general compactness criterion for equicontinuous mappings. The Arzelà theorem and the Ascoli theorem in C(X) spaces, X compact space. The Arzelà and the Ascoli theorems in C_0 (X) and $C_*(X)$ spaces, X locally compact space The Banach theorem on the weak* compactness of the closed balls of a separable Banach space.

4. FIXED POINT THEOREMS [6, 7]

Introduction to fixed point theorems. Banach fixed point theorem. Brouwer fixed point theorem. Compact mappings between normed spaces. Noteworty examples: Fredholm integral operators and Urysohn integral operators. Approximation of compact mappings by continuous mappings with finite dimensional range. The Schauder fixed point theorem. Some applications to integral equations and to ordinary differential equations (the Peano theorem). The Leray-Schauder principle and a priori estimates.

5. DENSITY THEOREMS [5]

Stone-Weierstrass type theorems for lattice subspaces and for subalgebras of C(X,IR) and C(X,C), X compact space. The Weierstrass theorems (algebraic form and trigonometric form). Quadrature formulas.
Stone-Weierstrass type theorems for subalgebras of $C_0(X,IR)$ e $C_0(X,C)$, X locally compact space. Application: the density in $C_0(IR^n,C)$ of the range of the Fourier transform on $L_1(IR^n)$.

6. POSITIVE LINEAR OPERATORS AND POSITIVE LINEAR FORMS ON CONTINUOUS FUNCTION SPACES, RADON MEASURES [4, 5]

Positive linear forms and positive linear operators on continuous function spaces. Positivity and continuity. Radon measures on locally compact spaces. Radon measures with finite support. The Riesz integral representation theorem. The dual space of $C_0(X,IR)$.

7. BOREL AND BAIRE MEASURES ON TOPOLOGICAL SPACES [3]

Stone vector lattices of functions. *F*-open subsets.The Daniell-Stone integral representation theorem. Baire and Borel σ-algebras and measures. *C(X,* **R**)-open subsets, C_b *(X,* **R**)-open subsets and ℵ *(X,* **R**)-open subsets. Examples of Borel and Baire measures: measures with densities, image measures, convolution products of measures. Regularity of Baire and Borel measures. The Lusin theorem

Vaguely convergent sequences of Baire measures. Vague convergence and stochastic convergence for distributions of random variables. Weakly convergent sequences of Baire measures. Vague convergence and weak convergence for probability Baire measures. The Poisson theorem. The Helly theorem and the Polia theorem for probability measures on IR. Vaguely compact subsets of measures. Discrete Baire measures and a relevant approximation theorem.

8. FOURIER TRANSFORMS OF MEASURES ON IR^n [3]

Fourier transforms of measures and functions on IR^n . The multiplication theorem. The uniqueness theorem. The continuity theorem of P. Lévy. Differentiability of Fourier transforms. Application: the Central Limit theorem.

9. KOROVKIN-TYPE THEOREMS AND POSITIVE APPROXIMATION PROCESSES [1, 2]

Korovkin-type approximation theorems in metric spaces. The classical Korovkin theorems (algebraic and trigonometric forms). Equivalence between the Korovkin theorems and the Weierstrass approximation theorems. Korovkin-type theorems for subsets of IR^n. Korovkin subsets in $C_0(X,IR)$, X locally compact space. Characterization of Korovkin subsets in terms of Radon measures. Application: Parabolic Korovkin subsets.

Positive approximation processes. The Bernstein operators, the Kantorovich operators, the Favard-Szasz Mirakjan operators, the Fejèr and the Poisson operators and their approximation properties. Approximation of continuous functions and L^p-functions in terms of polynomials. Cesàro summability and Abel summability of Fourier series of continuous 2π-periodic functions. The Dirichlet problem on the unit circle.

Teaching methods:
Lectures and exercise sessions.

Auxiliary teaching:

Assessment methods:
Oral exam

Bibliography:

[1] F. ALTOMARE, Korovkin-type Theorems and Approximation by Positive Linear Operators, Surveys in Approximation Theory, Vol. 5, 2010, 92-164, scaricabile gratuitamente presso http://www.math.technion.ac.il/sat/papers/13/, ISSN 1555-578X.

[2] F. ALTOMARE - M. CAMPITI, Korovkin-type Approximation Theory and its Applications, De Gruyter Series Studies in Mathematics, 17, De Gruyter & Co. Berlin, New York ,1994.

[3] H. BAUER, Measure and Integration Theory, De Gruyter Series Studies in Mathematics, 26, De Gruyter & Co. Berlin, New York, 2001.

[4] G. CHOQUET, Lecture on Analysis, vol. I, W.A. Benjamin Inc., New York, 1969.

[5] G. B. FOLLAND, Real Analysis, J. Wiley & Sons Inc., New York, 1999.

[6] K. GOEBEL, A concise course on fixed point theorems, Yokohama Publishers, 2002.

[7] E. ZEIDLER, Applied Functional Analysis, Vol. 109, Springer – Verlag, Berlin, 1995.