Xma$_{\text{th}}$s workshop

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Martino Borello (*École Polytechnique Fédérale de Lausanne*)
On the Euclidean minimum of number fields.

A number field K is called Euclidean with respect to the norm if its ring of integer O is a Euclidean ring with respect to the absolute value of the usual norm N from K to $\mathbb{Q}$. This means that for every x in K, there exists c in O such that $N(x-c) < 1$. This suggests to look at the positive real number

$$M(K) = \sup_{x \in K} \inf_{c \in O} N(x-c).$$

called the Euclidean minimum of K. If K is totally real, then Minkowski's conjecture [1] states that the inequality

$$M(K) \leq 2^{-n} \cdot \sqrt{d_K}$$

holds, where n is the degree of K and d_K is the absolute value of its discriminant. In 2005 [2] Curtis McMullen showed the topological core of Minkowksi's conjecture.
In the talk, I will present the history of the problem, giving some examples and showing what is known up to know. I will then sketch the methods of McMullen, showing a recent generalization to the context of number fields with mixed signature [3].

[1] H. Minkowski, Ueber die Annaherung an eine reelle Grosse durch rationale Zahlen, Math. Ann., 54: 91-124, (1900).
[2] C.T. McMullen, Minkowski's conjecture, well-rounded lattices and topological dimension, J. Amer. Math. Soc., 18: 711–734 (2005).
[3] E. Bayer-Fluckiger, M. Borello and P. Jossen, Inhomogeneous minima of mixed signature lattices, preprint 2015.

Biagio Cassano *("Sapienza" - Università di Roma)*
Hardy uncertainty principle and gaussian decay
for solutions to the Schrödinger equation.

We will discuss a version of the Hardy uncertainty principle for Schrödinger equations with external bounded electromagnetic potentials, based on logarithmic convexity properties of Schrödinger evolutions. Joint work with Luca Fanelli.

Salvatore de Candia *(Università degli Studi di Bari "Aldo Moro")*
Slant immersions.

The notion of slant submanifold in complex geometry was introduced by B.Y.Chen, in 1990, as a natural generalization of both holomorphic submanifolds and totally real submanifolds. Later, in 1996, A.Lotta introduced and studied the notion of slant submanifold of an almost contact metric manifold. After, many authors have investigated slant submanifolds immersed in an almost contact metric manifold whose underlying structure is cosymplectic or Sasakian or Kenmotsu. In this talk we briefly present some important results of this theory

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Davide Rossini (*Scuola Normale Superiore - Pisa*)
Localized Majorana-like modes in a number conserving setting.

The Kitaev chain is the simplest Hamiltonian example of one-dimensional fermionic model supporting interesting topological features [1]. Namely, an open-ended wire of length L possesses two nearly degenerate ground states with an energy difference that is exponentially suppressed with the size L. Such states are characterized by different fermionic parities and differ only for edge properties, which are well described by certain Majorana operators localized at the boundaries. Unfortunately, the Kitaev's Hamiltonian does not commute with the particle number operator. This requirement would be crucial in view of the possible experimental realizations with ultracold atoms loaded in optical lattices. Here we fill this gap and introduce an exactly solvable model of interacting fermions in a two-wire geometry [2]. Our model describes a topological superconductor supporting non-local zero-energy Majorana-like edge excitations and retains the fermionic number as a well-defined quantum number. The construction of the Hamiltonian with local two-body interactions and of its ground state draws inspiration from recent works on dissipative state preparation for ultracold atomic fermions [3], here applied to spinless fermions.

[1] A. Y. Kitaev, Phys. Usp. 44, 131 (2001).
[2] F. Iemini, L. Mazza, D. Rossini, S. Diehl, and R. Fazio, Phys. Rev. Lett. 115, 156402 (2015).
[3] S. Diehl, E. Rico, M. A. Baranov, and P. Zoller, Nature Phys. 7, 971 (2011).

Antonio Macchia *(Philipps-Universität Marburg, Universidade de Coimbra)*
The poset of proper divisibility.

We study the proper divisibility of monomials as an order relation in $\mathbb{N}^n$. This gives rise to a new class of posets. Surprisingly, the order complexes of these posets are homologically non-trivial. We prove that these posets are dual CL-shellable, we completely describe their homology (with integer coefficients) and we compute their Euler characteristic by using generating functions. Moreover we provide the first example of a non-CL-shellable poset whose dual is CL-shellable.
This is a joint work with D. Bolognini, E. Ventura and V. Welker.

Francesco Ferrulli *(Imperial College London)*
Spectral properties for bi-layer Graphene operator.

The aim of this brief talk is to present some established properties of the spectrum of the 1-D Dirac operator and introduce the 2-D operator which describes the Hamiltonian of a bi-layer Graphene system.

Giancarlo Garnero *(Università degli Studi di Bari "Aldo Moro")*
Moving Walls and Geometric Phases.

We unveil the existence of a non-trivial Berry phase associated to the dynamics of a quantum particle in a one dimensional box with moving walls. It is shown that a suitable choice of boundary conditions has to be made in order to preserve unitarity. For these boundary conditions we compute explicitly the geometric phase two-form on the parameter space. The unboundedness of the Hamiltonian describing the system leads to a natural prescription of renormalization for divergent contributions arising from the boundary.

Konstantin Palagachev *(University of the Federal Armed Forces at Munich)*
Value function approach for bilevel optimal control problems.

We investigate bilevel optimal control problems (BOCP), in which both the upper and lower level problem are infinite dimensional optimization problems. Exploiting the value function of the lower level problem, the original BOCP can be reduced to a single level optimal control problem. Techniques from nonsmooth analysis are applied, in order to obtain necessary optimality conditions of Pontryagin type. Applications to pursuer-evader and scheduling problems are considered.

Antonio Lerario (*Florida Atlantic University and SISSA*)
Statistics on Hilbert's Sixteenth Problem.

If we pick a real polynomial of degree d, the number of real zeroes can range anywhere from zero to d (but the number of *complex* zeroes, counted with multiplicity is exactly d). Similarly, the *complex* zero set of a generic real polynomial in two variables is a Riemann surface (a complex curve) whose topology depends only on the degree of the polynomial, but the number of components of its *real* part (a curve in the real plane) strongly depends on the coefficients of the polynomial (genericity only guarantees smoothness). Hilbert's Sixteenth Problem asks to study the "number, shape, and position" of these real components – a complete answer is known only for curves of degree up to 8! An exciting point of view comes by replacing the word "generic" with "random", asking for the typical situation. In this talk I will present a probabilistic approach to this problem, discussing recent developments on the topology of random manifolds.

Giuseppe Vacca *(Università degli Studi di Bari "Aldo Moro")*
Divergence Free virtual elements for the Stokes problem on polygonal meshes.

In the present talk we develop a new family of Virtual Elements for the Stokes problem on polygonal meshes. By a proper choice of the Virtual space of velocities and the associated degrees of freedom, we can guarantee that the final discrete velocity is pointwise divergence-free, and not only in a relaxed (projected) sense, as it happens for more standard elements. Moreover, we show that the discrete problem is immediately equivalent to a reduced problem with less degrees of freedom, thus yielding a very efficient scheme. We provide a rigorous error analysis of the method and several numerical tests, including a comparison with a different Virtual Element choice.

Salvatore Stuvard *(University of Zurich)*
Regularity theory of area-minimizing generalized surfaces.

In many problems arising in Analysis and Geometry, one is naturally led to deal with objects which, a priori, are allowed to exhibit singular behaviours. Nonetheless, at a later stage some of them turn out to be more regular than expected. A common technique to explore eventual regularity properties is the so called "blow-up analysis", that is the analysis of the "tangent objects" that our initial ones look like when put under a microscope. The focus of the talk will be Plateau's problem, that is the problem of minimizing the surface area for fixed boundary: I will present the modern approach in the framework of integer rectifiable currents and discuss the regularity properties of the minimizers. I will stress the importance of the analysis of the cone-like local structure of area-minimizing surfaces to obtain a satisfactory description of their singular set, and finally discuss some of the main open problems in the field.

Pierpaolo Vivo (*King's College London*)
Universality of Tracy-Widom distribution:
from solitaire card games to burning paper sheets.

The Tracy-Widom (TW) distribution was originally discovered in the realm of random matrices as the limiting distribution of the largest eigenvalue of Gaussian matrices. It is given by a complicated expression involving an integral of a function satisfying a second order-differential equation of the Painlevé type. In recent year, this distribution has cropped up in a number of seemingly unrelated problems, but the reason for this rather astonishing 'universality' still remains elusive. I will present the original context where TW was initially obtained, as well as two other examples: the statistics of the longest increasing subsequence of random permutations (the Ulam problem), and growth models in the KPZ universality class.

Fabio Pizzichillo *(BCAM - Basque Center for Applied Mathematics)*
Delta-Shell interaction for Dirac Operator and Klein's Paradox.

We consider the Dirac Operator coupled with a scaled short-range potential and we take the delta functions limit. We obtain different results with respect to the case of the free Dirac Operator coupled with an electotrostatic shell potential located on the boundary of a smooth domain. We explain how this discrepancy is related to the so called "Klein's Paradox".

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