5^{th} Workshop
Structural Dynamical Systems: Computational Aspects
SDS2008

Hotel Villaggio Porto-Giardino

Capitolo, Bari, Italy

17-20 June 2008

The Structural Dynamical Systems Workshop has been organized by *Dipartimento di Matematica, Università degli Studi di Bari* and *Dipartimento di Matematica, Politecnico di Bari*.

The members of the organizing committee are

- Luciano Lopez (Università degli Studi di Bari)
- Tiziano Politi (Politecnico di Bari)
- Nicoletta Del Buono (Università degli Studi di Bari)
- Elia Cinzia (Università degli Studi di Bari)

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- IMACS : International Association for Mathematics and Computers in Simulation

Contents

1 Introduction

The aim of the workshop is to discuss recent developments in computational methods for:

- numerical methods for ODEs;
- non-smooth dynamical systems;
- dynamical systems with variable structure;
- smooth decomposition of matrices;
- Krylov methods for matrix functions in ODEs.

This workshop, which follows four previous editions held in 2001, 2003, 2005 and 2006 consists of invited lectures and contributed talks on the above topics. Furthermore, SDS2008 will give the opportunity for deep discussions between participants and speakers.

We wish to thank all the collaborators who have contributed to this workshop and, in particular, Mario di Bernardo, (Università di Napoli, Italy), Luca Dieci (Georgia Institute of Technology, Atlanta, U.S.A.), Uwe Helmke (Universität Würzburg, Germany), Valeria Simoncini (Università di Bologna, Italy), Alessandro Spadoni (Georgia Institute of Technology, Atlanta, U.S.A), Erik Van Vleck (University of Kansas, Lawrence, U.S.A.), Elio Usai (Università di Cagliari).

Special thanks go to Marina Popolizio, Marco Berardi and Alessandro Pugliese for all their help and assistance.

The Organizing Committee.

2 Scientific and Social Program

The plenary and contributed talks will be arranged as described in the following table.

Time	Tuesday 17	Wednesday 18	Thursday 19	Friday 20
08:50 - 09:00	Opening address: L. Lopez			
09:00 - 09:45	Van Vleck	Simoncini	Dieci	Usai
09:45 - 10:30	Simoncini	Van Vleck	Usai	Dieci
10:30 - 11:00	*Coffee Break*	*Coffee Break*	*Coffee Break*	*Coffee Break*
11:00 - 11:45	Spadoni	Kuepper	di Bernardo	di Bernardo
11:45 - 12:15	Pugliese	Lessard	Guglielmi	Cherubini
12:15 - 12:45	Papini	Breda	Buric	Paternoster
12:45 - 14:30	*Lunch*	*Lunch*	*Lunch*	*Lunch*
14:30 - 15:00	Elia	Spaletta	Sofroniou	
15:00 - 15:30	Poster Session	Sgura	*Excursion*	
15:30 - 16:00	Poster Session	Diele	*Excursion*	
20:30			*Social Dinner*	

3 Invited Lectures

- **Mario di Bernardo**: Talk 1: Discontinuity-induced bifurcations of equilibria in piecewise-smooth dynamical systems.
  Talk 2: Discontinuity-induced bifurcations of limit cycles in piecewise-smooth dynamical systems.

- **Luca Dieci**: Talk 1: Systems with Discontinuous Right-Hand Sides: Examples, Sliding Modes, Fundamental Matrix.
  Talk 2: Sliding Motion in Flippov Systems: Theory and Numerics.

- **Valeria Simoncini**: Approximation of functions of large matrices: computational aspects and applications.

- **Alessandro Spadoni**: Eigenvalue Coalescence and energy gaps in dynamic systems of engineering interest.

- **Erik Van Vleck**: Talk 1: The Error in Orthogonal Integration: Theory.
  Talk 2: The Error in Orthogonal Integration: Applications.

- **Elio Usai**: Talk 1: Sliding modes: basic theory and new perspectives.
  Talk 2: Applications of Sliding Mode theory in science and engineering.

Mario di Bernardo

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Discontinuity-induced bifurcations of equilibria in piecewise- smooth dynamical systems

This talk will be concerned with the investigation of discontinuity- induced bifurcations of equilibria and fixed points in PWS flows and maps. After presenting a brief classification of the systems of interest, attention will be focused to all those phenomena occurring whener the fixed point or equilibrium of a PWS system crosses the boundaries in phase space where the system is discontinuous. The classification strategy first proposed by Feigin will be outlined and illustrated through some representative examples.

Discontinuity-induced bifurcations of limit cycles in piecewise-smooth dynamical systems

Following from Talk 1, the case of DIBs of limit cycle will be discussed with particular attention to the case of impacting and Filippov systems. The classification method based on the use of so- called discontinuity maps will be presented and illustrated through some representative examples: cam-follower devices in Mechanics and Power Converters in electronics.

Luca Dieci

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Systems with Discontinuous Right-Hand Sides: Examples, Sliding Modes, Fundamental Matrix.

In this talk we review concepts from the theory of differential systems with discontinuous right-hand sides (switching systems). We particularly emphasize sliding modes and review the concept of "Fundamental Matrix Solution" associated to these systems. We illustrate interesting questions on these systems by means of a few examples.

This is a joint work with Luciano Lopez from the Department of Mathematics, University of Bari.

Sliding Motion in Flippov Systems: Theory and Numerics.

In this talk we present some results on Filippov systems. In particular, we discuss second order corrections to the Filippov theory, and propose a novel way to define the sliding vector field on the intersection of two or more surfaces. We also present a numerical method and illustrate its performance on examples.

This is a joint work with Luciano Lopez from the Department of Mathematics, University of Bari.

Valeria Simoncini

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Approximation of functions of large matrices: computational aspects and applications

The problem of numerically approximating the action of the matrix function $f(A)$ on a vector v for a given matrix A is of great importance in a wide range of applications. Over the years, several methods have been proposed for approximating the action of matrix functions such as the exponential, rational, sign and trigonometric functions. More recently, great attention has been paid to the computational aspects of this problem when large dimensions occur, due to the possible advantages of using matrix function evaluations over standard discretization techniques. In fact, the use of matrix functions is the core of many exponential integrators for solving systems of ordinary differential equations or time-dependent partial differential equations.

When A has large dimension, the computation of $f(A)v$ may be effectively approximated by projecting the problem onto a subspace of possibly much smaller dimension. The Krylov subspace

$$K_k(A, v) = \text{span}\{v, Av, \ldots, A^{k-1}v\}$$

has been extensively used to this purpose, due to its favourable computational and approximation properties.

In the first talk, we discuss several aspects associated with an efficient and accurate computation of $f(A)v$, such as stopping criteria and acceleration procedures.

In the second talk, we discuss theoretical and practical implications of these strategies in the numerical solution of related problems, such as the Lyapunov matrix equation.

Alessandro Spadoni

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Eigenvalue Coalescence and energy gaps in dynamic systems of engineering interest.

Spatially or frequency-confined phenomena in dynamic systems foreshadow both potentially catastrophic events as well as great opportunities to manipulate a system's response. Generally, the intrinsic characteristics of a dynamic system, regardless of its surrounding conditions, are established via eigenvalue analyses, whether the system's response be of transient or steady nature. Among the myriad phenomena that can be studied through eigenvalue analysis, the focus of this presentation is the coalescence of eigenvalue pairs as well as zones of eigenvalue repulsion. In the first case, eigenvalue-pair coalescence is often accompanied by their rapid divergence, known as eigenvalue loci veering, a catastrophic event. In the latter scenario, eigenvalues repel each other producing what is known as band gaps, or regions where momentum transfer is forbidden. Such attributes of the eigenvalues of a dynamic system, in fact, determine whether energy entrapment will take place, for a spatially-confined response, or lack of momentum transfer, for a frequency-confined response. The conditions necessary for eigenvalue-loci veering and band gaps are discussed from an engineering standpoint. Examples of dynamic systems exhibiting either band gaps or eigenvalue coalescence are presented.

Erik Van Vleck

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The Error in Orthogonal Integration: Theory.

For the general case of integration of a matrix equation of the form

$$\dot{Q} = f(t, Q), \;\; Q(t_0) = Q_0 \text{ orthogonal}$$

for the solution to remain orthogonal $Q^T \dot{Q}$ must be skew-symmetric, hence $f(t, Q) = Q\ S(t, Q)$ where $S(t, Q)$ is skew-symmetric. In this talk we consider the particular case in which the orthogonal solution is a change of variables that brings a linear time dependent ODE to upper triangular. Our focus is on the global error in approximating the time dependent orthogonal change of variables. Bounds on the global error are obtained that depend on the local error in approximating Q, the non-normality in the upper triangular factor, and on integral separation, a natural condition for characterizing spectral gaps in the time dependent case.

The Error in Orthogonal Integration: Applications.

Applications and implications of having global error bounds for the orthogonal change of variables will be presented on the approximation of Lyapunov exponents and Sacker-Sell spectrum for ODEs and for PDEs on inertial manifolds, the existence of ground states for some linear asymptotically hyperbolic Schrodinger operators, and the embedding of dissipative lattice differential equations in inhomogeneous reaction-diffusion PDEs. We will elaborate on extensions to discrete time dynamical systems, unitary change of variables for linear time varying ODEs with complex coefficient matrix functions, and applications to the approximation of the Evans function, an important tool for determining stability of traveling wave solutions.

Elio Usai

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Sliding modes: basic theory and new perspectives

Considering dynamical systems, sliding modes can appear when the system trajectory, i.e., a solution of the differential equation describing the system motion, is constrained to belong to a specific surface or manifold. During the sliding mode the "velocity" vector defining the system dynamics is characterised by being directed towards the constraint, i.e., the sliding surface; this also means that the system dynamics switches abruptly and at very high frequency, theoretically infinite, so that the system behaviour on the sliding surface is different from any unconstrained motion of the system.
This peculiar property attracted the interest of researches in the automatic control area who developed a specific control approach based on Variable Structure Systems with Sliding Modes. In this talk the main basic features and properties of Variable Structure Systems with Sliding Modes are presented and discussed, considering some of the most interesting aspects of the sliding mode theory both from the mathematical and "logic" point of view. In particular the concept of "equivalent control" and of "Filippov's solution" will be presented and related to the problem of finding a specific solution of a differential inclusion. The stability analysis of "classic" Sliding Modes will be presented by means of the well know Lyapunov approach to the stability of the equilibrium.
Then the extension of the Sliding Mode concept to integral manifolds is presented with the introduction of the so-called Higher-Order Sliding Modes. Considering the switching function as the system output, classical Sliding Modes can be referred to as First-Order Sliding Modes since discontinuity appears in the first derivative of the switching function, while in Higher-Order Sliding Modes the discontinuity appears in a higher derivative of the switching function. The main algorithms implementing Higher-Order Sliding Modes will be presented and their properties discussed.
Finally, some problems arising from the discrete-time implementation of the algorithms forcing Sliding Mode behaviours will be analysed.

Applications of Sliding Mode theory in science and engineering.

In this talk some applications that are of interest in engineering and mathematics are presented. When considering engineering, most applications of the Sliding Mode theory are in Automatic Control since the Variable Structure Control with Sliding Modes presents very interesting invariance properties which improve the robustness of the control system with respect to both external disturbance and modelling uncertainties. In this framework an example of application of Sliding Mode Control to a constrained mechanical system will be given considering the contact force regulation in high-speed pantographs. Nevertheless, Sliding Mode theory can be also used to analyse the performance of some specific electronic devices such as the 1-bit converters, i.e., a Delta-Sigma modulator . Some specific control problems such as state estimation and unknown input reconstruction can be seen as dynamic system inversion problems, which it is well known to be hill-posed problems. The Sliding Mode approach allows for their regularization by integration, so that real-time algorithms can be implemented. By resorting to the features of Sliding Modes some algorithms implementing robust and exact differentiators will presented and their features of accuracy and insensitivity with respect to measurement noises are discussed. Similarly to the derivative estimation, some other typical mathematical problems can be re-formulated as control problems. Simple ideas about the solution of non linear equations and the extremum seeking will be presented in the framework of the Sliding Mode Control theory.

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4 Contributed Talks

- **Breda Dimitri**: Numerical stability analysis of infinite-dimensional dynamical systems.
- **Buric Lubor**: Traffic flow as a Filippov dynamical system.
- **Cherubini Anna Maria**: An asymptotic study for a collision model.
- **Diele Fasma**: Symplectic partitioned Runge-Kutta methods for optimal growth models.
- **Elia Cinzia**: Numerical techniques for exponential dichotomy.
- **Gugliemi Nicola**: A regularization of discontinuous differential equations.
- **Kuepper Tassilo**: Reduction to invariant cones for non-smooth systems.
- **Lessard Jean-Philippe**: Forcing theorems in dynamics: a computational approach.
- **Papini Alessandra** Software for 1-D smooth SVD continuation.
- **Paternoster Beatrice**: New classes of two step collocation methods for special second order ODEs.
- **Pugliese Alessandro**: Coalescing singular values: Theoretical results and algorithms.
- **Sgura Ivonne**: A reaction-diffusion model for electrochemistry: analytical, computational and experimental issues.
- **Sofroniou Mark**: Stiffness Detection Revisited.
- **Spaletta Giulia**: Effective composition integrators for ODEs.

Dimitri Breda

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Numerical stability analysis of infinite-dimensional dynamical systems

The long-time behaviour of the trivial solution of linear (or linearized) dynamical systems of infinite dimension is often determined by the point-spectrum of suitable infinite-dimensional operators acting on a Banach space. Such operators are either represented by the associated semigroup of solution operators or by its infinitesimal generator whenever this latter can be defined.

Under the rather general assumption of eventual compactness of the semigroup, the spectra of these operators are made of only isolated points. These characteristic values can then be computed by using pseudospectral techniques, ending with numerical schemes whose convergence is of infinite order (spectral accuracy). These methods provide discretizations of the operators and thus reduce the original problem to that of computing the eigenvalues of a matrix for which standard algorithms can be applied.

In this talk we present the main features of the pseudospectral approaches based on the discretization of either the infinitesimal generator or the solution operators semigroup. Local asymptotic stability properties of wide classes of linear autonomous retarded and partial retarded functional differential equations can be analyzed via the infinitesimal generator approach. Also age-structured population dynamics modeled by hyperbolic partial differential equations enter this frame. Finally, similar dynamical systems but with periodic coefficients can be treated via the solution operator approach by discretizing the associated monodromy operator.

This is a joint work with Stefano Maset from Department of Mathematics, University of Trieste, and Rossana Vermiglio from Department of Mathematics and Computer Science, University of Udine.

Lubor Buvrivc

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Traffic flow as a Filippov dynamical system

We consider a classical microscopic follow-the-leader model of the traffic flow. The model simulates the behavior of N identical cars on a circular road of the length L. It is well known that the model breaks down at the time instant when two cars collide. Nevertheless, the natural action of a driver at that moment would be to overtake the slower car. In [1] we proposed a model of overtaking. We investigated oscillatory solutions (patterns) to the overtaking model.

Assuming $N = 3$ as a case study, we managed to formulate the overtaking model as a Filippov system i.e., ODEs with discontinuous right-hand side. We exploit the Filippov system formulation to continue the oscillatory patterns with respect to the length L of the road. In the present contribution we generalize the Filippov system formulation of the overtaking model assuming $N \geq 3$ cars on the road. We give some simulation results with a particular attention to new oscillatory patterns.

This is a joint work with Vladimír Janovský from Charles University, Faculty of Mathematics and Physics, Sokolovská 83, 18675 Prague 8, Czech Republic.

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Anna Maria Cherubini

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An asymptotic study for a collision model

This is a joint work with G. Metafune and F. Paparella from Universita' del Salento - Dip. Matematica

The simplest and most widely used model of a bouncing ball (or grains of a granular fluid) assumes that the ball is a rigid body, and that an impact with the floor is an instantaneous event, which reverses the vertical component of the speed of the ball: to model energy dissipation caused by an impact, it is customary to introduce a positive coefficient of restitution $r < 1$, which is the ratio between the absolute values of the vertical speeds immediately after and before an impact.

This model performs well when the ball does not experience too many impacts in the time unit, otherwise granular systems described in this way are generically subject to the phenomenon of *inelastic collapse*, where clusters of particles undergo an infinite number of collisions in a finite time: after this time these models are meaningless.

Models using restitution coefficients rest on hypotheses that become invalid as the frequency of the impacts diverges: collisions with the floor are not truly instantaneous, and treating the ball as indeformable is highly questionable when the frequency of impacts is close to the resonant frequencies of the bouncing ball.

In our model of a bouncing ball we explicitly take into account the deformability of the body and we give up the notion of restitution coefficient, at least as a primitive concept; we still assume impacts to be instantaneous, but only from a microscopic point of view: a persistent contact with the floor is seen as a rapid sequence of instantaneous impacts.

The idealized ball is seen as two point masses connected by a massless dissipative spring; impacts are modeled as an instantaneous elastic collision between the lower mass and the floor.

We prove that our model is free from pathologies analogous to the inelastic collapse and compute the asymptotic expression for inpact times and for the impact velocity. We also prove that contacts with zero velocity of the lower end of the ball are possible, but non-generic; finally we prove that starting from any initial condition, the system tends to the static equilibrium as $t \to \infty$.

Our results are fully exposed in

A.M. Cherubini, G. Metafune, F. Paparella, *On the Stopping Time of a Bouncing Ball*, DCDS B (10) Number 1, July 2008, 48-72

Fasma Diele

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Symplectic partitioned Runge-Kutta methods for optimal growth models

We are concerned with the discretization of optimal control problems, in both finite and infinite time horizon case, when a Runge-Kutta scheme is selected for the related Hamiltonian system. It is known that deriving order conditions for the state-costate equations corresponds to checking for order conditions for a symplectic partitioned Runge-Kutta scheme. In the present talk this result is extended to growth models, where the system is described by a current Hamiltonian, widely used in economics studies. We demonstrate that the current state-costate system requires a Lawson exponential scheme for the costate approximation.

This is a joint work with C. Marangi from Istituto per le Applicazioni del Calcolo 'M. Picone', CNR, Via Amendola 122/D, 70126 Bari, Italy (c.marangi@ba.iac.cnr.it) and S. Ragni from Facoltà di Economia, Università di Bari, Via Camillo Rosalba 56, 70100 Bari, Italy, (s.ragni@ba.iac.cnr.it).

Cinzia Elia

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Numerical techniques for Exponential Dichotomy

We propose numerical techniques to establish exponential dichotomy on the whole line. To do so together with the exponential dichotomy on the positive and negative half line, we will also need to approximate stable and unstable subspace. An application to the orbital stability of travelling waves will be presented as well.

This is a joint work with Luca Dieci from Georgia Institute of Technology and Erik Van Vleck from Univeristy of Kansas.

Nicola Guglielmi

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A regularization of discontinuous differential equations

We consider a regularization for a class of discontinuous differential equations arising in the study of neutral delay differential equations with state dependent delays. For such equations in fact the possible discontinuity in the derivative of the solution at the initial point may propagate along the integration interval giving rise to subsequent points, called breaking points, where the solution derivative is still discontinuous. As a consequence, in a right neighbourhood of each such point we have to face a Cauchy problem where the equation has a discontinuous right-hand side. In this case the existence and the uniqueness of the solution is no longer guaranteed to the right of such points and hence the solution of the neutral equation may either cease to exist or bifurcate. The regularization is based on the replacement in the r.h.s. of the derivative of the solution by its time average over an interval of length ε and then considering the limit as $\varepsilon \to 0^+$. We show that the regularization we consider allow us to define a weak global solution on bounded time intervals, which agrees with Filippovs definition. Several properties of the solutions corresponding to small values of $\varepsilon > 0$.

Tassilo Kuepper

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Reduction to invariant cones for non-smooth systems

The reduction of smooth dynamical systems to lower dimension center manifolds containing the essential bifurcation dynamics is a very useful approach both for theoretical investigations as well as for numerical computation. Since this approach relies on smooth new properties of the system and on the existence of a basic linearization the question arises if this approach can be carried over to non-smooth systems. Extending previous works we show that such a reduction is indeed possible by using an appropriate Poincaré map: the linearization will be replaced by a basic piecewise linear system; a generalized fixed point of the Poincaré map generates an invariant cone which takes the role of the center manifold. The occurrence of nonlinear higher order terms will change this invariant "manifold" to a cone-like surface in $\mathbb{R}^n$ containing the essential dynamic of the original problem. In that way the bifurcation analysis can be carried out for a one-dimensional map.

Jean Philippe Lessard

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Forcing theorems in dynamics: a computational approach.

In this talk, we introduce a new forcing theorem which has application in proving the existence of chaotic dynamics for a given class of ODEs. In particular, we prove that the stationary Swift-Hohenberg equation has chaotic dynamics on a critical energy level for a large (continuous) range of parameter values. The first step of the method relies on a computer assisted, rigorous, continuation method to prove the existence of a periodic orbit with certain geometric properties. The second step is topological: we use this periodic solution as a skeleton, through which we braid other solutions, thus forcing the existence of infinitely many braided periodic orbits. A semi-conjugacy to a subshift of finite type shows that the dynamics is chaotic.

Alessandra Papini

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Software for 1-D smooth SVD continuation

In last years we have been working on the construction of a code for the computation of smooth SVD paths for matrix functions depending on a real parameter. Assuming well separated singular values all along the path, it is easy to produce smoothly varying factors of the SVDs. The task becomes numerically more difficult when two singular values get close to coalescence. In this situation, a robust stepsize selection strategy is crucial. This involves several issues: computation of accurate predictors for the SVD factors, Procrustes techniques to correct factors computed by canned software, zoom-in procedures to locate crossing or near-crossing of singular values. In this talk, we will review the main features of our code and present a few examples.

This is a joint work with Luca Dieci from Georgia Institute of Technology, Maria Grazia Gasparo from Università degli Studi di Firenze, and Alessandro Pugliese from Università degli Studi di Bari

B. Paternoster

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New classes of two step collocation methods for special second order ODEs

We introduce some families of two step collocation methods for second order Ordinary Differential Equations (ODEs) of type $y'' = f(x, y)$, having periodic or oscillatory solutions. In the construction of the collocation function, which provides a continuous approximation to the solution of the ODEs, for second order ODEs different posssibilities can be taken into account. First we have to choose if we want to approximate also the derivative of the solution, in addition to the solution in the step points, as for instance Runge-Kutta-Nyström methods do in the one step case. Then we can use also stage values which are associated to the previous step points, in order to highten the order of the method, without higthening the computational cost too much [2]. We extend the multistep collocation technique presented in [3]-[4], in order to derive new two step General Linear Methods [2] for second order ODEs. We provide some examples of two step hybrid collocation methods [1], analyze their order and stability properties. We also consider the possibility of collocating on basis of functions other than algebraic polynomials.

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This is a joint work with R. D'Ambrosio from Dipartimento di Matematica e Informatica, University of Salerno and M.Ferro from Dipartimento di Matematica e Applicazioni, University of Naples "Federico II"

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Coalescing singular values: Theoretical results and algorithms

In this talk, we consider real matrix functions that depend on two parameters and study the problem of how to detect and approximate parameters' values where the singular values coalesce. First, we introduce some theoretical results connecting the existence of coalescing points to the periodic structure of the smooth singular values decomposition computed around the boundary of a domain enclosing the points. These results form the backbone of our algorithms for the detection and approximation of coalescing points in planar regions. Finally, we present techniques for continuing curves of coalescing singular values of matrices depending on three parameters, and illustrate how these techniques can be used to locate coalescing singular values of complex-valued matrices depending on three parameters.

This is a joint work with Luca Dieci from Georgia Tech.

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A reaction-diffusion model for electrochemistry: analytical, computational and experimental issues

To understand the formation of morphological patterns found in electrodeposition (ECD), we introduce a reaction-diffusion system that can describe the coupling between surface morphology and surface composition.

We investigate the nonlinear dynamics of the system from the analytical and numerical points of view. The stability analysis shows the initiation of spatial patterns induced by diffusion, i.e. the *diffusion-driven* or *Turing instability phenomenon*. Moreover, from phase space analysis, the existence of travelling waves (kink) is proved. Computational aspects related to the numerical approximation of the solutions by high order finite difference techniques are discussed.

We present some numerical simulations that are in good agreement with experiments for the electrodeposition of Au-Cu alloys.

This is a joint work with
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Stiffness Detection Revisited

Many applied differential equations exhibit some form of stiffness which restricts the step-size and hence effectiveness of explicit solution methods. A number of implicit methods have been proposed to circumvent this problem. However implicit methods can also be substantially less efficient, because of the overhead associated with the intrinsic linear algebra. Several attempts have been made to provide user friendly codes that would automatically attempt to detect stiffness at run-time and switch between appropriate methods as necessary. In this talk we will survey the strategies that have been proposed to automatically equip a code with a stiffness detection device. Particular attention will be given to the problem of estimation of the dominant eigenvalue of a matrix. We will illustrate a new implementation, based on subspace iteration, that appears to be very promising. Finally, numerical experiments will be given to demonstrate the effectiveness of the new strategy.

This is a joint work with Giulia Spaletta from Bologna University, Italy

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Effective composition integrators for ODEs

This work focuses on the derivation of composition methods for the numerical integration of ordinary differential equations. Composition is a useful technique for constructing high order approximations whilst conserving certain geometric properties. We survey existing composition methods and describe results of an intensive numerical search for new methods. Details of the search procedure are given along with numerical examples which indicate that the new methods perform better than previously known methods.

This is a joint work with Mark Sofroniou from Wolfram Research, Champaign, Illinois, U.S.A.

5 Posters

- **Hany Hosham Bakit** Reduction to an invariant two-dimensional ”manifold” for a six-dimensional non-smooth Brake-system.
- **D’Ambrosio Raffaele**: Development and Implementation of Two-step Runge-Kutta Methods for Ordinary Differential Equations.
- **Nicoletta Del Buono**: Gradient flow approaches for orthogonal non-negative matrix factorization.
- **Ferro Maria**: Twostep RungeKutta methods for Ordinary and Stochastic Differential Equations.
- **Garrappa Roberto**: Numerical approximation of a generalized Mittag–Leffler matrix function
- **Messina Eleonora**: Comparing analytical and numerical solution of two delays integral equations.

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Reduction to an invariant two-dimensional ”manifold” for a six-dimensional non-smooth Brake-system

Summary

Following the general ideas to reduce non-smooth systems to invariant cones generalizing center manifolds we investigate a six-dimensional brake system. The only equilibrium at the origin and for important case where both matrices have a complex eigenvalues. Using the Poincar map we establish the existence of cone-like invariant manifolds. Which will be used to discus the generation of periodic orbits and their stability.

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Development and Implementation of Two-step Runge-Kutta Methods for Ordinary Differential Equations

It is the purpose of my PhD thesis, drawn up under the joint supervi- sion of Prof. Beatrice Paternoster (University of Salerno) and Prof. Zdzis- law Jackiewicz (Arizona State University), to develop new special classes of TwoStep RungeKutta methods (TSRK) for the numerical resolution of Ordinary Differential Equations. We show constructive techniques of both continuous TSRK methods and discrete ones, in order to achieve high order and stage order of convergence and strong stability properties. Using continuous approximants, we achieve reliable error estimations, in or- der to develop a variable stepsizevariable order implementation strategy. We also consider TSRK methods with Inherent Quadratic Stability (IQS), i.e. TSRK methods with quadratic stability function, extending the idea of the Inherent RungeKutta Stability developed for GLMs, in order to show an alghoritmic technique which allows us to practically construct TSRK methods with strong stability properties. In particular, we derive the class of one point spectrum TSRK methods with IQS

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Gradient flow approaches for orthogonal non-negative matrix factorization

Non-negative matrix factorization (NMF) arises from many important applications such as multivariate data analysis, image and spectral data processing, text mining, environmental science, neural learning process, sound recognition, chemometric, object characterization, DNA gene expressions. In general, the problem is the following: given a matrix Y of "observed" data, where the elements of the matrix to be analysed are non-negative, find reduced rank non-negative matrices Q and S so that

$$Y \approx QS.$$

Depending on the peculiar application is going to be tackled, in addition to low-rank and non-negativity, other conditions need to be imposed on the factors Q and S. Some of these constraints include sparsity, smoothness, specific structures and so on. Particularly, we are going to emphasise the orthogonality of matrix factors in NMF. We will review a number of numerical techniques to solve to orthogonal non-negative matrix factorization problem and we also develop some new numerical methods based on the gradient approach.

This is a joint work with Simone Fiori from Dipartimento di Elettronica, Intelligenza Artificiale e Telecomunicazioni, Università Politecnica delle Marche.

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Twostep RungeKutta methods for Ordinary and Stochastic Differential Equations

The basic idea of my PhD thesis is the construction, the analysis and the implementation of new numerical methods for Ordinary Differential Equations (ODEs) and Stochastic Differential Equations (SDEs) having high order and strong stability properties. Twostep RungeKutta methods, already considered in the context of ODEs, and not yet used in the numerical integration of SDEs, seem to be promising, because the presence of extra parameters can raise the order of convergence and improve the stability properties, without increasing the computational cost. For ODEs, the constructive technique is the collocation extended to the multistep case, [2],[3],[4]. As far as SDEs [1] are concerned, the aim is to produce a first formulation of twostep stochastic RungeKutta methods.

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Numerical approximation of a generalized Mittag–Leffler matrix function

This work deals with the computation of a vector of the form $F_\gamma(A)v$, where $F_\gamma(z)$ is a *generalized Mittag–Leffler* function, A a real matrix and v a real vector.

Problems of this kind arise in the discretization of partial differential equations with boundary conditions perturbed by a fractional noise [2] and as special cases in conflict–controlled processes for systems with fractional derivatives in game theory [1].

Krylov subspace methods are commonly employed to evaluate matrix functions; by following the approach investigated in [3], we propose an algorithm which combines Krylov subspace methods with recent results about Padé rational approximations to $F_\gamma(z)$ presented in [4].

Numerical experiments seem to show the effectiveness of the proposed technique with respect to other viable approaches.

This is a joint work with Marina Popolizio from the Department of Mathematics, University of Bari, Italy.

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Comparing analytical and numerical solution of two delays integral equations

We consider a model of age-structured population dynamics based on Volterra integral equations where the kernel is a discontinuous function and, in particular, it is zero outside a bounded interval depending on some age constants (the maturation and the maximum age). This results in a Volterra integral equation with two constant delays. To our knowledge, a numerical study of this kind of equations is still undeveloped in the literature. Thus, our goal is to construct and analyze suitable numerical methods for this kind of equations. With this aim we have tuned classical numerical methods to the special form of the problem and studied their convergence. Furthermore, we have investigated on the stability properties of these methods through the analysis of significative test equations (the basic, the convolution and a nonlinear test equation) and found sufficient, but also necessary and sufficient conditions for the numerical solution to behave like the analytical one.
This is a joint work with E.Russo (University of Naples) and A.Vecchio (Ist. per Appl. del Calcolo "M.Picone", Naples - CNR).

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