

# ICTMT 11

11th International Conference on Technology in Mathematics Teaching



Bari, 9<sup>th</sup> – 12<sup>th</sup> July 2013

Department of Mathematics - University of Bari

## Conference Proceedings

edited by

**Eleonora Faggiano & Antonella Montone**



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## **Plenary Keynote Speakers**

Gilles Aldon, * cole Normale Sup rieure de Lyon, France*  
Regina Bruder, *University of Darmstadt, Germany*  
Michal Yerushalmy, *University of Haifa, Israel*  
David Murrels, *Secondary Teacher, England*  
Ketty Savioli, *Primary Teacher, Italy*

## Preface

The Eleventh International Conference on Technology in Mathematics Teaching took place from Tuesday July 9<sup>th</sup> to Friday July 12<sup>th</sup>, for the first time in Italy, at the University of Bari, in collaboration with the University of Torino.

As in the spirit of the ICTMT Conferences, it aimed to bring together lecturers, teachers, curriculum designers, mathematics researchers, learning technologists and educational software designers, who shared an interest in improving the quality of teaching and learning by an effective use of technology.

Digital technologies are now becoming ubiquitous, so nobody can ignore them. It is extremely urgent to analyse their influence on the education systems and to explore and pave the way for other possible research.

The Conference aimed therefore to be a forum in which researchers and practitioners could exchange and discuss better practices, theoretical know-how, innovation and perspectives on educational technologies and their impact on maths teaching and learning.

Five plenary speakers were involved to broaden the focus of the conference. Michal Yerushalmy, arguing that digital books offer new kinds of flexibility, participation, and personalization, speculated about challenges for a new textbook culture. Regina Bruder, describing and analysing some results of the project CALiMERO, pointed out that, apart from the availability of technology and suitable teaching and learning materials, the training of didactical-methodical competencies of the teachers in this field is crucial for the efficient technology-based teaching. The third keynote has been devoted to the experience of a secondary school teacher and a primary school teacher. The former, Dave Murrels, explained how he had become increasingly aware of the interdependence of some key factors upon the successful use of ICT in his classroom and, looking forward, why his long term aim is for students to develop their mathematical thinking skills through, and eventually, independently of the technology. The latter, Ketty Savioli, reflected on the potential of motion detectors when used to introduce the concept of function with children, suggesting that if we want to be able to exploit the potential success of the integration of technology in school mathematics teaching, the search for why(s) is a (not only didactical) enterprise that is still our responsibility. Gilles Aldon, finally, underlined that the observation of a changing world where “digital natives” have a different perception of things is often made but the question of how to change teaching and learning in order to adapt schools and society is an open question. He suggested a

methodology, the incident analysis, to analyze the modifications of class dynamics and to better understand the students and teacher's joint action.

Last but not least, more than sixty delegates (coming from 25 different Countries) exposed and discussed their papers (in forms of oral presentations, workshops or posters) on the following themes:

- *Curriculum*, The impact of technology on the mathematics curriculum
- *Assessment*, New possibilities for assessment in mathematics
- *Students*, Technology to motivate and support students to learn mathematics
- *Teachers*, Technology for mathematics teachers' professional development
- *Innovation*, New development in technology for learning and teaching mathematics
- *Applications*, Technology as a bridge between mathematics and other subjects
- *Software*, Design, evaluate and choose software to learn and teach mathematics

The high level of papers submitted has been guaranteed by the International Scientific Committee. Two peer reviewers have examined and accepted all different proposals.

This book is gratefully dedicated to all the Scientific Committee Members and Conference delegates. Special thanks have to be given to the Organizing Committee and all the people who have contributed to the success of the Conference.

Eleonora Faggiano and Antonella Montone

*Department of Mathematics  
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## Plenary Keynotes

Aldon G. HOP-O'-MY-THUMB AND MATHS EDUCATION

*In the new world of information and communication new students come into classrooms and universities with a different relation with knowledge. Teachers' roles become more brokers' roles leading to use and to give sense to knowledge much more than transmitting it. Different didactical tools take into account the new paradigm of teaching; particularly, the documentational genesis which is interested in the process of acquisition and transformation of resources into the teachers' documentational system of resources. On an other hand, the notion of didactic incidents should give tools to analyze the modifications of class dynamics and to better understand the students and teacher's joint action.*

Bruder R. CAS USE IN SECONDARY SCHOOL MATHEMATICS - TEACHING STYLE AND MATHEMATICAL ACHIEVEMENT

*Report on the basic ideas and elements of a teaching concept for the CAS-use starting in grade 7 in the project CALiMERO (2005-2010) and results of an evaluation study on this project. On the basis of aggregated data from lesson journals correlations between elements of the teaching concept achieved in the different learning groups and performance evaluated in the tests are analyzed.*

Murrels D. EVALUATING THE IMPACT OF GRAPHING SOFTWARE ON LEARNING AND STUDENTS MATHEMATICAL THINKING

*The main aim of this action research project was to evaluate the impact of investigative tasks involving the use of the dynamic graphing software Autograph v3.2 upon students' learning and mathematical thinking. A range of research tools and instruments were created and used to generate and analyse both quantitative and qualitative data. The results show a wide range of impact upon students' mathematical thinking and show which areas of mathematical thinking are most impacted upon. Viewing the results through the themes of other research into the use of graphing software and theories of algebraic thinking allows for critical reflection upon the authors current and future practice.*

Savioli K. AN EXPERIENCE AT PRIMARY SCHOOL: GRAPHING MOTION IN THE MATHEMATICS CLASSROOM

*In this paper I discuss a longitudinal study on graphing motion at primary school through the aid of motion detectors. I took part in the study as a teacher of a class of 16 children that was involved in the study from grade 2 to 5. Referring to some instances from written protocols produced by the children or from actions*

*in the classroom, I reflect about the potential of motion detectors when used to introduce the concept of function with children, and whether we can say that the motion detectors “work” in the mathematics classroom and, especially, what they work or do not work for. I will also discuss the other side of the study, which is more concerned with my teaching practice and children’s engagement.*

Yerushalmy M.      LEARNING MATHEMATICS WITH DIGITAL TEXTBOOKS:  
CHALLENGING AUTHORING AND AUTHORITY

*A textbook is a special type of book that is part of institutionalized schooling, usually used in a particular way. Traditionally it has a specific structure and is designed to contain the message of the professional community about what students should learn. It also represents the ideas of the author about how the content should be taught and learned. Digital books offer new kinds of flexibility, participation, and personalization – properties that are in contrast to the traditionally authoritative structure of the textbook and the passiveness of the reader. To study the implications of the proposed affordances of digital textbooks, this paper explore three scenarios, analyzing the challenges involved in each and speculating about challenges for a new textbook culture.*

## Oral Presentations

Abar C.                                    PEDAGOGICAL STRATEGIES TO TEACH AND LEARN  
MATHEMATICS WITH THE USE OF GEOGEBRA

*This paper describes, in the researches carried out between 2009 and 2011 in the Postgraduate Studies Program - Mathematics Education using GeoGebra, which theoretical and methodological supports were used, GeoGebra's role in each of them, the considerations about its use, the goals of each author, and the results obtained. Considering aspects of the thematic analysis, it was found that out of all nineteen papers, nine of them used different theories and are analyzed in this study. The use of theoretical supports can be considered as an alternative to find ways to develop situations using GeoGebra that help to overcome common difficulties in teaching and learning mathematics. The conclusions show that the authors could search, throughout the entire strategy, the best solution for the activities proposed in a context that accepts the inclusion of traditional and digital media.*

Albano G.,                                MATHEMATICS EDUCATION AND ELEARNING: MEANINGFUL  
Ferrari P. L.                              USE OF THE QUIZ-MODULES

*In this paper we discuss the use of close-ended questions in self-training and formative self-assessment in mathematics e-learning. Besides the undeniable value of automatic feedback and assessment, we argue how to avoid drawbacks of this tool offered by e-learning platforms, and focus on how to exploit the potential of this tool also by taking into account some key features of mathematics learning, such as multisemioticity. We give some indications on how to construct effective close-ended questions apt to actually evaluate competencies and not only contents.*

Arzarello F.,                              WAYS OF MANIPULATION TOUCHSCREEN IN ONE  
Bairral M., Danè C.,                    GEOMETRICAL DYNAMIC SOFTWARE  
Yasuyuki I.

*Touchscreen devices are spreading and becoming familiar for many students. This research identifies ways of screen touching during the process of solving problems using the Geometric Constructor (GC) software. We designed a case study with some Italian High School students, who used tablets to solve geometric problems. Since manipulation on tablet is different from that with mouse clicking, this kind of research investigates a new aspect of students' behaviours when using dynamic geometry software. Based on researches by Yook (2009) and Park (2011), we provide a new schema focused on geometrical thinking and strategies used by students to solve the proposed tasks. We observed singularities in the way*

*students rotate (using one or more than one finger) and a different way of dragging: the dragging to approach.*

Arzarello F.,                   THE PLANIMETER AS A REAL AND VIRTUAL INSTRUMENT  
Manzone D.                    THAT MEDIATES THE INTRODUCTION OF AREA

*The paper describes a particular approach to the notion of area in the secondary school as “swept area”, using a specific old professional tool, the planimeter. It allows measuring the area of a plane domain basing exactly on the idea of swept area. Hence students are introduced to a fresh notion of area, which is intrinsic of a figure and is not strongly based on the idea prevailing in the school, which founds the notion of area on algorithms, possibly confusing these with the concept itself. Using the instrument, the students are introduced into a didactical cycle that structure their learning processes. A semiotic lens is used to describe it.*

Barzel B.                        ESTABLISHING CONDITIONS FOR COMPUTERALGEBRA AS AN  
  ADDITIONAL VALUE IN MATHEMATICS TEACHING

*There is an ongoing debate about chances and burdens of implementing Computeralgebra in the classroom. Schools, school administration and especially mathematicians at universities are engaged in this open debate and still a lot of countries do not involve Computeralgebra in their examinations and mathematics classrooms. On one side the main reasons for excluding Computeralgebra from classrooms can be summarized as apprehension that students loose paper-and-pencil-skills such as solving an equation by hand. On the other side the reasons for including Computeralgebra into the classrooms are shown by a lot of studies which describe the potential of Computeralgebra in teaching and learning mathematics such as learning a better algebraic insight. Against this background it is a big challenge for school administrations to decide in a correct and sustainable way to give schools proper and clear principles for their teaching and learning at schools. This challenge was the reason that in 2010 the Ministry of Thuringia in Germany commissions a meta-study to find out conditions that ensure a successful use of Computeralgebra in teaching mathematics and in assessment. This meta-study, written for the ministry, is the content of this presentation and paper.*

Bray A.,                         THE HUMAN CATAPULT AND OTHER STORIES - ADVENTURES  
Oldham E.,                     WITH TECHNOLOGY IN MATHEMATICS EDUCATION  
Tangney B.

*This paper reports on-going research into how the affordances of off-the-shelf technologies can be aligned with relevant mathematics pedagogy, to create transformative learning experiences with the potential to overcome some of the well-known impediments to mathematics teaching and learning. From a systematic analysis of recent literature on digital technologies and mathematics*

*education, a set of guidelines has been formulated by two of the authors for the design of innovative and engaging interventions in mathematics education. In this paper the guidelines are presented, along with results from experiences with two such interventions. An exploratory case study methodology is employed, and the paper reports on an initial pilot study, the results of which suggest that the interventions are pragmatic to implement and may improve affective engagement and motivation.*

Brown M.,                      THE MATHEXPLORER SYSTEM: STUDENT EXPLORATION WITH  
Steel C.D.C.                    A MATLAB-BASED SYSTEM

*This article describes an approach currently being carried out at the University of Manchester where students are encouraged to explore and appreciate the mathematics as a complement to the traditional-style lectures. 1st and 2nd year students in Electrical Engineering use various ‘notebooks’ making use of Mupad in order to explore some mathematical topics, working faster than can be done when every calculation is by hand and gain an appreciation of the wider concepts surrounding the topics and how they fit into the appropriate topics within Electrical Engineering. The notebooks can be used during individual student study, or in teams, or can be used as part of lecture demonstrations.*

Button T.,                      FURTHER PURE MATHEMATICS WITH TECHNOLOGY: A POST-  
Lee S.                            16 UNIT OF STUDY THAT USES TECHNOLOGY IN THE  
TEACHING, LEARNING AND ASSESSMENT

*Further Pure Mathematics with Technology is a new optional A level Mathematics unit that can be taken by pre-university students in England. The unit has been developed by Mathematics in Education and Industry, a mathematics education charity. It requires students to have access to technology, in the form of a graph-plotter, spreadsheet, programming language and computer algebra system (CAS) for the teaching, learning and assessment. This paper describes the development of the unit, including the rationale for the design decisions, and the implications for future developments of this type.*

Chiappini G.                    CULTURAL AFFORDANCES OF DIGITAL ARTEFACTS IN THE  
TEACHING AND LEARNING OF MATHEMATICS

*The last two decades have seen a great development of research on the use of technology in teaching and learning of mathematics but, to date, these studies have not had a significant influence on teaching practice because research was unable to take sufficiently into account the challenges that digital technology poses in this context of use (Joubert, 2013). These challenges mainly concern the use of technology to facilitate reasoning in mathematics and to facilitate the construction of meanings, principles, values that are rooted in the historical,*

*cultural development of this discipline. This contribution aims to develop a methodology centred on the concepts of affordance and narrative for describing and evaluating the potential of educational mediation of an artefact for the indicated objectives. The article is based on a previous work by Chiappini (2012) and the observations to this work carried out by Monaghan and Mason (2012).*

Clark-Wilson A.,           CORNERSTONE MATHEMATICS: AN APPROACH TO  
Hoyle C., Noss R.       TECHNOLOGY-ENHANCED CURRICULUM INNOVATION AT  
SCALE

*We report on a project, Cornerstone Mathematics, that is researching the impact and scalability of an innovation for secondary mathematics, focused on embedding digital technology at points where its potential for enhancing the learning of mathematics is clear due to the availability of multiple representations that are visual, dynamic and inter-connected. The innovation combines four elements, each of which has been extensively researched: digital technology designed for mathematics learning; professional development; new curriculum 'replacement' materials; and explicit strategies established for scaling and sustainability. We will present the results of a pilot study in 19 English classrooms with students aged 11-13 years that reported positive outcomes, supported by the evidence from measured gains between pre- and post-testing, teachers' and students' feedback, and structured observations of experimental lessons.*

Costabile F. A.,           A PROGRAMMING ENVIRONMENT AS A METHODOLOGICAL  
Serpe A.                 TOOL FOR THE LEARNING OF MATHEMATICS

*This work focuses on the use of technology in the classroom for the teaching-learning of mathematics. After a brief outline of the current situation in Italian schools, we address the issue of programming as an important teaching tool for its pedagogical and educational value. The central part of this study stresses the importance of choosing a programming environment suited to the teaching context. A practical teaching example and some conclusions round off the paper.*

Daher W.,                 PRE-SERVICE TEACHERS' PERCEPTIONS OF THE INTEGRATION  
Baya'a N.                OF ICT IN THE MATHEMATICS CLASSROOM

*This research examines middle school mathematics pre-service teachers' professional development during two years of their study at a teacher college regarding their perceptions of ICT use in their teaching of mathematics, specifically as a result of their preparation in ICT integration in the mathematics classroom in the frame of two didactic courses (in their second year of study) and in the frame of practicing teaching mathematics with technological tools (in their third year of study). The results indicated that generally, the pre-service teachers, as a result of their two years preparation, perceived the importance of integrating diverse*

*technological tools in the mathematics classroom, probably because of the visual aspect of mathematics that the technological tools enable, which helps students with their learning of mathematics and encourage them to do so because they are part of their lives.*

Demir O., Heck A.      A NEW LEARNING TRAJECTORY FOR TRIGONOMETRIC  
FUNCTIONS

*Educational research has shown that many secondary school students consider the subject of trigonometric functions as difficult and only develop shallow and fragmented understanding. It is unclear which of the two popular approaches to introducing trigonometry, namely the ratio method and the unit circle method, works best. In this study we propose a new framework for trigonometric understanding and a new, dynamic geometry supported trajectory for learning trigonometric functions. We also report on the results of a classroom case study in which the new approach has been implemented and researched. We discuss the task-related difficulties that students faced in their concept development and we describe their trigonometric understanding in terms of our framework.*

Faggiano E.,              USING INTERACTIVE WHITEBOARDS TO ENHANCE MATHS  
Montone A.,              TEACHING: HOW, WHEN AND WHY?  
Ancona R. L.

*This paper focuses on how interactive whiteboards (IWBs) can be used to enhance maths teaching and learning, the attention being especially concentrated on the role and fundamental qualities of teachers. Special attention is also paid to the skills of teachers so as to make maximum use of these tools. It has also been stressed how a correct use of IWBs can be functional, i.e. how useful these tools can be when students approach, develop and strengthen mathematical concepts.*

Ferrara F.,              INVENTIVE MOMENTS TO MOBILISE SINUSOIDAL FUNCTIONS  
De Simone M.

*This paper presents ways of animating the concept of function at primary school, through the aid of a motion detector that works with bi-dimensional motion. The technological environment favours a graphical approach to function, engaging the children in bodily activities. Starting from the new materialist perspective offered by de Freitas and Sinclair, we want to discuss ways of moving, doing and knowing in the classroom as pedagogical possibilities of in(ter)vention and inventiveness to mobilise the mathematical concepts at play. Particular focus will be on the instance of sinusoidal functions and their relative properties through reference to circular motion.*

Ferrarello D.,            USING DGS TO INTRODUCE ALGEBRA PROPERTIES  
Mammana M. F.,  
Pennisi M.

*This paper describes a summary of an experimental study that involved teachers and students in 10 high-schools in Sicily together with a team of University Professors. The topic of the study was on symbolic algebra. The goal was to investigate the impact on teachers and learners of an “innovative” way of introducing algebra inspired by a modern reinterpretation of the second book of Euclid’s Elements, and based on the use of a dynamic geometry system. The activity has been carried out in 15 classes of the first years of high-school (14-15 year-old students). Results of the experimentation are provided.*

Hirsch C.                AN EXTENDED DESIGN EXPERIMENT CONNECTING  
SOFTWARE DEVELOPMENT WITH CURRICULUM  
DEVELOPMENT: THE CASE OF THE CORE-PLUS MATHEMATICS  
PROJECT

*In the United States and internationally, design and development of mathematical and statistical software most often occurs independently of the development of school mathematics curricula. In this chapter we describe an extended design experiment that has pursued an integrated design approach that connects software development with the development of a curriculum spanning the full range of contemporary high school mathematics. The resulting software, CPMP-Tools, is an open-source suite of linked Java-based software tools that include general purpose tools—a spreadsheet, a computer algebra system, dynamic geometry, data analysis, simulation tools, and discrete mathematical modeling tools—together with topic-focused custom apps.*

Jackiw N.                TOUCH & MULTITOUCH IN DYNAMIC GEOMETRY:  
SKETCHPAD EXPLORER AND "DIGITAL" MATHEMATICS

*This paper documents potential impacts of the novel multitouch tablet screen, popularized by Apple’s iPad, on Dynamic Geometry software use, design, and research. Work with Sketchpad Explorer relates multitouch technology to the representation and control of multiple variables within the mathematical environment; to the conditions and expressions of embodied forms of cognition at the juncture between the environment and its user; and to the possibilities of multi-user technological and pedagogic interaction within the large social context of use.*

Jiang Z., White A.,     INVESTIGATING THE IMPACT OF A TECHNOLOGY-CENTERED  
Sorto A.,                TEACHER PROFESSIONAL DEPARTMENT PROGRAM  
Rosenwasser A.

*This study investigated the impact of a technology-centered professional development program on high school geometry teachers' change and their students' geometry learning. 64 geometry teachers were randomly assigned to two groups. The teachers in the experimental group participated in a one-week summer institute followed by six half-day workshop sessions during the school year, in which they studied the critical features of the dynamic geometry (DG) approach and the DG-oriented teaching strategies they were expected to use in their classrooms. The teachers in the control group received workshop sessions of mathematics content and taught as before. Teachers in both groups were found to be faithful to the instructional approaches assigned to them. Teachers in the experimental group scored higher in a conjecturing and proving test than did teachers in the control group. The students of teachers in the experimental group significantly outperformed the students of teachers in the control group in a geometry achievement test.*

Keisoglou S.,                    MEDIATED MEANINGS FOR THE CONSTRUCTION OF THE  
Lymperopoulou E.,        CONE IN A 3D DIGITAL ENVIRONMENT  
Kouletsi E.

*In this paper, we explore the ways 10th grade students organize their mathematical knowledge and the meanings created in the construction of the right circular cone through their engagement in three different tasks. We want to study whether the resources of the medium, a 3d Logo / Turtle Geometry environment, 'Machine Lab Turtleworlds' (MaLT), which combines multiple representations of geometrical objects and their dynamic manipulation, mediate a construction by students who are taught solely the Euclidian constructions, having available only the compass and protractor. The results illustrate a progression in the process of the construction of new meanings in mathematics and development of student's focusing onto the objects within the setting, as different areas of mathematics are interwoven, marked by shifts in analyzing and interpreting the mathematical notions engaged in the tasks.*

Lo M.,                            MATHPEN: IDENTIFYING AND SOLVING THE PROBLEMS OF  
Edwards J-A.,                ONLINE COLLABORATIVE LEARNING FOR MATHEMATICS  
Bokhove C.,  
Davis H.

*Combining the interactive communication power of Web 2.0 and social-constructivist theory in education research, online collaborative learning (OCL) has now become an area of intensive research and has generated many favourable results. Yet, the term online collaborative learning, or any other related terms, are seldom seen in mathematics education journals. This paper will, after a brief*

*overview of OCL theory, describe the problems associated with OCL in mathematics education and offer MathPen (an online handwriting recognition system) as a potential solution.*

Lymperopoulou E., THE RECONSTRUCTION OF MEANING FOR THE CONGRUENCE  
Doukakis S. OF TRIANGLES WITH TURTLE GEOMETRY

*In the present study, 'Turtleworlds' a programmable Turtle Geometry medium, is utilized by teachers and students as a means of exploring the congruence of triangles with the help of a half-baked microworld. This activity brought to light a relation which, somehow, brings order to chaos, creating categories (classes) of triangles, each one represented by the unique, constructed triangle. It was also shown that in order to arrive at the process of proof as documentation, it is essential to begin with a generalization of observations, formulation of arguments and their articulation in unified reasoning, so as to enable the student to understand and effectively formulate more formal proofs.*

Maclaren P. THE NEW CHALKBOARD: THE ROLE OF DIGITAL PEN  
TECHNOLOGIES IN TERTIARY MATHEMATICS TEACHING

*Mathematics is a discipline with a distinctive pedagogy that reflects how knowledge is expressed and developed in symbolic and diagrammatic form. Pedagogical approaches have both influenced, and been influenced by, the architectural design and educational technologies of the environments in which they are used. In the tertiary sector in particular, traditional methods of teaching mathematics have been challenged by computing technologies that are based on keyboard and mouse interfaces and learning environments that emphasise digital displays. This Paper discusses how the use of pen-enabled Tablet PCs can build on the benefits of traditional pedagogical approaches while facilitating the development of new approaches.*

Marshall N., EXPLORATORY OBJECTS AND MICROWORLDS IN UNIVERSITY  
Buteau C., Muller E. MATHEMATICS EDUCATION

*This paper is centred in university mathematics education. It draws parallels between the work of students who develop, program and use Exploratory Objects, a requirement in a core mathematics program at Brock University, and the work of students with microworlds as it is reported in the literature. In both of these computer environments our lens is on the students' activities as they develop and program them, and our focus is on students' potentially learned skills. This work highlights a fundamental gap between research and sustained implementation of student generated computer environments by university mathematics majors.*

Maschietto M.,           THE BEGINNING OF THE ADVENTURE WITH PASCALINE AND  
Soury-Lavergne S.       E-PASCALINE

*The paper presents the idea of “duo of artefacts”, constituted by the pascaline (i.e., the arithmetical machine Zero+1) and its digital version e-pascaline. This “duo of artefacts” is proposed here to support student’s learning about the position notation in base ten at primary school. It also represents an example in which the development in technology (Cabri Elem environment) allows the relationships between material and virtual manipulatives to be discussed.*

Miranda M.,           LACK OF SENSE OF PURPOSE IN THE USE OF TECHNOLOGY  
Sacristán A. I.         FOR MATHEMATICAL TEACHING

*The use of technology as a support for teaching-learning processes has grown exponentially within educational systems worldwide, and Mexico has not been the exception. We report on how teachers, from several schools within three important high-school level educational systems in this country, perceive, understand and use digital technologies (DT) as a support in their practice, for what aims and in which ways. It has been proposed that adequate use of DT can achieve meaningful learning in students; however, in our research, we found that teachers, as well as institutions, lack clarity and knowledge on how to integrate digital tools to improve mathematical learning in students.*

Miyazaki M.,           A WEB-BASED LEARNING SYSTEM FOR CONGRUENCY-BASED  
Jones K., Fujita T.     PROOFS IN GEOMETRY IN LOWER SECONDARY SCHOOL

*International research confirms that many secondary school students can find it difficult to understand and construct mathematical proofs. In this paper we report on a research project in which we are developing a web-based learning support platform (available in Japanese, English and Chinese) for students who are just starting to tackle congruency-based in geometry in lower secondary school. In using the technology students can complete the congruency-based proofs by dragging sides, angles and triangles to on-screen cells and our system automatically translates the figural elements to their symbolic form. Using the notion of ‘conceptions of congruency’ as our framework, we compare the tasks provided in our web-based learning system with similar tasks in a typical textbook from Japan. Our analysis shows that the tasks provided in the web-based platform aim to help learners to develop a correspondence conception of triangle congruency.*

Nicaud J.,             DYNAMIC ALGEBRA IN EPSILONWRITER: A PEDAGOGICAL  
Maffei L.             PERSPECTIVE

*Dynamic Algebra means ‘doing calculations with the mouse’. A Dynamic Algebra*



*We discuss here the collaborative aspect of online explorations of videos of free-fall phenomena, by five adult students enrolled in a continuing education course on “Physics and its algebra”. These explorations are part of an ongoing research project on promoting mathematical learning via the process of building math models in a context of rich experimentation and virtual collaboration in an online environment. The project includes various independent, constructionist explorations connected to different mathematical and physical ideas. The specific tasks presented here, involved discussions on previous knowledge about free-fall phenomena, taking measurements from videos, plotting and building mathematical models. But the focus of this paper is to illustrate the role of the participants’ discussions thru the project’s social network (that included forums, blogs and chat), to strengthen their internal constructions of the mathematics associated with free-fall phenomena.*

Paiva J., Amado N., PEER FEEDBACK IN THE CONTEXT OF A CONSTRUCTION TASK  
Carreira S. WITH GEOGEBRA

*In this paper we present an episode of a school year teaching experience with 7th grade students where it was privileged the work with the computer in the teaching and learning of mathematics. We intend to discuss and analyse the nature of coactions between the computer and students and peer feedback, when solving a triangle classification task in pairs using GeoGebra.*

Pihlap S. THE IMPACT OF COMPUTER USE ON LEARNING OF  
QUADRATIC FUNCTIONS

*Studies of the impact of various types of computer use on the results of learning and student motivation have indicated that the use of computers can increase learning motivation, and computers can have a positive or a negative effect, or no effect at all on learning outcomes. Some results indicate that it is not computer use itself that has a positive or negative effect on the achievement of students, but the way in which computers are used. This study explores the impact of computer use on learning quadratic functions in the ninth grade. The study involves five classes where computers are used alongside traditional methods and five classes with only traditional forms of learning. There are no significant differences in learning outcomes between the students who use computers and those who do not; the students who use computers have higher motivation for learning functions compared with those who do not use computers.*

Robotti E., DIFFICULTIES IN ALGEBRA: NEW EDUCATIONAL APPROACH  
Fernando E. BY ALNUSET

*This paper discusses the difficulties students encounter in algebra, considering in*

*particular those students affected by dyscalculia. It is generally ascertained that dyscalculic students have difficulties arithmetic but, according to a strictly didactic point of view, we will try to answer the following questions: How they can approach algebraic manipulation and how they can grasp the meaning of algebraic manipulation? The aim of this paper is to present a qualitative analysis of the potentiality of a new software of dynamic algebra named AlNuSet, which favours not only a axiomatic-deductive approach to algebra but also a dynamic-perceptive one. The devised perspective has been based on the research developed in the domain of mathematics education and it has been integrated with the resulting data of the fields of psychology and of the neuro-sciences. The analysis presented in this report is a part of a work in progress, which involves students at the Upper Secondary School.*

Ronchi P.,                      GEOGEBRA E-LEARNING LAB  
Sargenti A.

*Since its foundation in 2010, the GeoGebra Institute of Turin has proposed training projects and experimental teaching methodologies to Italian teachers. In the same year the GeoGebra Institute of Bari was founded and immediately embarked on a close working relationship with its partner in Turin. The reasons for this collaboration can be found not only in the need to have an equal standard in the certificates that institutes provide, but above all because of their shared research background and their experience in teacher training. The keywords that characterize the experience are: project-community-practice. In the light of the experience gained by both institutes, we present the 'Geogebra eLearning Lab' using a Moodle platform which we will develop, where a different type of learning experience called "a Learning Event" takes place. These activities are designed to stimulate the participants to think and to interact with each others, as well as to produce further activities and resources.*

Sinclair N.                      TOUCHCOUNTS: AN EMBODIED, DIGITAL APPROACH TO  
LEARNING NUMBER

*This paper describes the design of a digital technology focussed on early number sense (especially counting and adding). This "TouchCounts" application (designed for the iPad) takes advantage of the easily shareable, multimodal touchscreen interface that provides direct mediation through fingers and gestures. After describing the affordances of the application and its relation to current literature on the role of fingers in the development of number sense. Using a new materialist theoretical lens, I analyse the way in which a group of four 3-4 year old preschool children become fluent with cardinal aspects of number.*

Štěpánková R.,           COMPUTER SIMULATION IN SOLVING PROBLEMS FROM  
Tlustý P.                   PROBABILITY

*Probability is part of school mathematics which students are often afraid of it. Not infrequently it is taught by using formulas and without great clarity. Teachers do not usually use computers for simulation of random chances in the theory of probability in the Czech Republic. Students rarely come across some example which should be solved by computer simulation. Although there are exceptions. We would like to show that computer simulation can play an important role in understanding of secondary school students - for example while solving problems from random walk. In this case, students can use stochastic graph. Then it is possible to build a formula for the solution of the problem. In our lessons, we have seen that for a lot of students it is not illustrative enough to be it. They can't interpret it because they don't understand the graph. At that time computer simulation has the irreplaceable role. It gives an illustrative and more comprehensible solution (or a way to solve it) of this problem.*

Tripconey S.,           THE IMPACT OF TRAINING COURSES ON MATHEMATICS  
de Pomerai S.,         TEACHERS' USE OF ICT IN THEIR CLASSROOM PRACTICE  
Lee S.

*Are teachers more likely to incorporate ICT in their teaching if they have done (a) a training course that is specifically devoted to exploring ICT packages to use in the classroom or (b) if they have done a training course that incorporates ideas for using ICT as part of a course focused on developing specific subject knowledge and pedagogy? As a significant provider of professional development for post 16 mathematics teachers in England, the Further Mathematics Support Programme reflects on the impact of their training courses on teachers' use of ICT in the classroom.*

Tsistos V.,           TRANSITIONS BETWEEN MICRO-CONTEXTS OF  
Stathopoulou C.,     MATHEMATICAL PRACTICES: THE CASE OF ARC LENGTH  
Kouletsis E.

*The study presented here is concerned with transitions between micro-contexts of mathematical practices. These micro-contexts are determined by the use of different software and material. The main focus is on a task in which students had to participate in investigating activities related to the arc-length. The task completion demonstrates a productive interaction of tool use which combines "instrumental approaches" to achieve the given purpose. At the same time it provides a framework for the observation of knowledge and skills transfer during transitions from one micro-context to another.*

Turgut M.                      APPLICATIONS MATHEMATICA INTO TEACHING OF LINEAR ALGEBRA: THE CASE OF LEAST-SQUARES

*In recent years, it can be seen from the available literature, not only various studies on the teaching of calculus appear but also a considerable number of studies on the teaching of linear algebra are presented by researchers. According to results, linear algebra has always been a challenge for learners and teachers. At this point, researchers agree on the use of educational technologies in the teaching of linear algebra. In this work, a teaching design of a special topic “least-squares” by the aid of Mathematica® software is developed. The tasks are formulated according to teaching of linear algebra frameworks. The steps of the design (use of matrices, plotting data, least-squares lines and curves) are explained in detail. Further research proposals are also posed.*

Turgut M., Uygan C.    SPATIAL ABILITY TRAINING WITH 3D MODELLING SOFTWARE

*In this study, activities supported by SketchUp, a 3D modelling software which is used to improve the spatial ability of primary school students have been developed. These activities have been developed considering the exemplary applications in the 6th-8th grade mathematics curriculum in Turkey and also in technical literature related to spatial ability training. The activities include “unit cubes” for the 6th grade students and “surface closure” for the 8th grade students. The unit cube activity aims to improve students’ ability to visualise structures given in different positions; the surface closure activity aims to improve students’ ability to visualise the surface development of basic geometric objects.*

Weigand H.                      FLASHING BACK AND LOOKING AHEAD - DIDACTICAL IMPLICATIONS FOR THE USE OF DIGITAL TECHNOLOGIES IN THE NEXT DECADE

*Advantages and disadvantages of the use of digital technologies (DT) and especially of computer algebra systems in mathematics lessons are discussed controversially worldwide. What will be the meaning of DT in the next years or even the next decade? The basis of the following considerations is a long-term empirical project  $M^3$  (Model-project New Media in Mathematics lessons) which was started ten years ago in 2003 to test the use of symbolic calculators (SC) in Bavarian “Gymnasien” (grammar schools) in Germany. In 2013 there exists a widespread experience in the use of SC in the grades 10 to 12 in classroom activities, student and teacher documents as well as test and examination results of students. The implications of this project are going to be focused in 10 theses or hypotheses of possible, gainful developments in the future. These theses will be explained with examples from the project  $M^3$ .*

## Workshops

Aldon G.,                   DESIGNING RESOURCES FOR TEACHER EDUCATION WITH  
Barzel B.,                   TECHNOLOGIES: EDUMATICS PROJECT  
Clark-Wilson A.,  
Robutti O.

*Using ICT such as spread sheets, geometry packages or computer algebra when learning and teaching mathematics is recommended or compulsory in the curricula of many European countries by good reasons. Technology offers the potentiality to enhance the learning of mathematics (Zbiek, 2007). But to develop this potentiality it is important to support teachers to come along with this challenge. This was the aim of the EU project EdUmatics (European development for the use of mathematics technology in classrooms). In the frame of this project researchers and teachers of ten countries worked together and developed systematically and theory-based an internet platform for teachers of mathematics in secondary schools and their teacher trainers with material for classrooms and professional development.*

Jackiw N.                   THE GEOMETER'S SKETCHPAD WORKSHOP: EXPLORING  
NON-EUCLIDEAN GEOMETRY WITH THE POINCARÉ DISK

*This paper briefly introduces a hands-on computer workshop offered at ICTMT11, and orients participants to reusable resources.*

Harris D.,                   USING DYNAMIC GEOMETRY FOR PROBLEM-SOLVING AND  
Mackrell K.                INQUIRY FROM 16-18

*This series of three workshops will focus on the theme of exploration, a central theme of the new International Baccalaureate (IB) mathematics courses that is also relevant to other mathematics curricula. Using dynamic geometry software (Cabri II Plus, Cabri 3D and Geometer's Sketchpad), participants may explore a set of recently developed resources designed to facilitate the learning of sequences and series, vectors, functions, and calculus through problem-solving and inquiry. Ways to use these resources in the classroom will be discussed, and participants will be shown how the resources may be modified and how selected resources were developed.*

Nicaud J.                   EPSILONWRITER: EDITING TEXT AND FORMULAS, DYNAMIC  
ALGEBRA, QUESTIONNAIRES AND COMMUNICATION WITH  
MATH

*Epsilonwriter is Java software running on Windows, MacOs and Linux, for editing text and formulas. Beyond allowing the production of documents and web pages*

*with a very flexible formula editor, Epsilonwriter implements Dynamic Algebra, an innovative and rich mechanism for step by step calculations. It includes a module for questionnaires (Multiple choice questions and questions with open mathematical answers). Epsilonwriter also has a chat, and allows working on Live documents (several persons share a document in real time). In all cases, math formulas are easy to type and are received as objects that can be edited.*

## Posters

Aldon G., EDUMATICS PROJECT: TEACHERS EDUCATION WITH  
Barzel B., TECHNOLOGIES  
Clark-Wilson A.,  
Robutti O.

*The EdUmatocs project aims to provide teachers of secondary mathematics with support to learn to use and integrate technology within their classrooms. The resources for professional development, whilst aimed at teachers, include a range of tasks for students to enable them to use technology within modeling and problem-solving activities. The resources include links to free and trial software, applications and animations in addition to task sheets and help sheets that can be adapted for different scenarios.*

Bauer A. REASONING WITH MULTIPLE AND DYNAMIC REPRESENTATIONS

*This paper presents a research in progress which aims to answer the question if learners working with digital multiple and dynamic representations really refer to the dynamics and the multiplicity in their arguments. In an empirical investigation learners were given mathematical reasoning tasks along with the appropriate representations, which were partially multiple, dynamical, or both. The hand-written documents were analysed regarding the appearance of multiple or dynamic representations in the learners arguments. Results indicate that the given representations have a great influence most of the time, but not always.*

Cazzola M., WIMS: A WWW INTERACTIVE MULTIPURPOSE SERVER  
Perrin-Riou B.,  
Reyssat E.

*In this poster we wish to introduce WIMS, showing its potentials as a tool for the teaching of mathematics (and beyond).*

Ismail N. INTERACTION: A KEY COMPONENT TO SUCCESSFUL ONLINE  
LEARNING

*E-learning, a process to create and provide access to learning when the source of information and the learners are separated by time and distance, has special kinds of barriers which need to be known and considered, for instance the high drop-out rate of e-learning and the suitability of e-learning to cover different subjects. Interactivity in e-learning is considered to be more than just clicking a mouse. This thesis argues that an in-depth understanding of interactivity in e-learning will reinforce and enhance the capabilities of this learning mode, and consequently*

*have a measurable positive impact on the aforementioned limitations.*

Oldenburg R.            THE GAINS AND THE PIFALLS OF DESIGNING EDUCATIONAL  
MATH SOFTWARE BY PRINCIPLES OF MATEMATICS

*This poster briefly describes two software prototypes developed by the author that address different aspects of the algebra taught in secondary schools. Both programs are designed to reflect specific aspects of algebra as a mathematical discipline.*

Surynková P.            RECENT ADVANCES IN THE APPLICATION OF 3D GEOMETRIC  
MODELING SOFTWARE WITH FOCUS ON LINEAR PERSPECTIVE

*A problem with difficulty of studying and understanding 3D geometry at secondary schools and colleges is addressed in this article. One possibility of improvement the understanding geometry is suggested - modeling with modern geometric and algebraic systems. The integration of computer modeling into the studying and teaching geometry is seems to be as efficient aid how to innovate the schooling of geometry and achieve better results. The application of geometric modeling software will be demonstrated on example of linear perspective - the central projection.*

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E. Faggiano & A. Montone (Eds)

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