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Contents

Asymptotic properties of some positive operators of Altomare

when where

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Primary AMS Classification: 41A36

Keywords and phrases: Approximation by Positive Operators, Rate of Convergence

We study the local rate of convergence for some sequences of operators recently defined by Professor Altomare and his collaborators. As a main result we derive the complete asymptotic expansions. Also simultaneous approximation is considered.

References

[1] F. Altomare and S. Milella, *Integral-type operators on continuous function spaces on the real line,* J. Approx. Theory 152(2) (2008), 107–124.

[2] F. Altomare and R. Amiar, *Approximation by positive operators of the C_0-semigroups associated with one-dimensional diffusion equations. I,* Numer. Funct. Anal. Optim. 26(1) (2005), 1–15.

[3] F. Altomare and Ioan Raşa, *On a class of exponential-type operators and their limit semigroups,* J. Approx. Theory, 135(2) (2005), 258–275.

[4] F. Altomare and A. Attalienti, *Degenerate evolution equations in weighted continuous function spaces, Markov processes and the Black-Scholes equation. II.,* Results Math. 42(3-4) (2002), 212–228.

[5] F. Altomare and E.M. Mangino, *On a class of elliptic-parabolic equations on unbounded intervals,* Positivity 5(3) (2001), 239–257.

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One Topics On Fast wavelet Algorithm For One -Dimensional Haar Wavelets

When, Where

Mahmoud Afshari, *Persian Gulf University* , *Iran*

Primary AMS Classification: 42C40

Keywords and phrases: Häar wavelets, Fast wavelets, Wavelet algorithm, Estimation density Function, Discreet wavelet, Multiresolution analysis.

Wavelets are regarded by many as primarily a new subject in pure and applied mathematics. perhaps one of the most common application of wavelets is in signal processing . In this paper We apply wavelets to estimate any signal under consideration by using a mathematical function $s = f(t)$. We consider a sample point (t_j, s_j) includes a value $s_j = f(t_j)$ at height s_j and abscissa (time or location) t_j. To analyze a signal or function in term of wavelets, we propose , obtaining a knew algorithm of wavelet decomposition by using one dimensional of Fast Häar wavelet. We present some relationship between wavelets coefficients for approximating any signal.

q-Approximation Processes on Unbounded Intervals

When, Where

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Primary AMS Classification: 41A36

Keywords and phrases: q-integers, weighted modulus of smoothness, statistical convergence

The approximation of functions by using linear positive operators introduced via q-Calculus is currently under intensive study. The pioneer work has been made by A. Lupaş and G.M. Phillips who proposed generalizations of Bernstein polynomials based on q-integers.

Recently, many research papers focused on the investigation of linear q-operators for the approximation of real valued functions defined on bounded intervals.

In this talk we will discuss q-analogues of different discrete type approximation processes for functions having a polynomial growth and defined on an unbounded interval. Both qualitative and quantitative results are established. We will see that various properties of the classical sequences are inherited by their q-analogues.

Among our outcomes we mention the following. The moments of the operators are explicitly expressed with the help of new q-analogue of Stirling numbers. By using a weighted modulus of smoothness, the rate of local and global convergence is given. In the frame of weighted spaces we also deal with the statistical approximation property of the involved operators. Variants of our sequences which preserve certain polynomials are introduced and we show that the order of approximation is at least as good as the order of the initial operators.

Spectral properties of matrix operators in Banach space

When, Where

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Primary AMS Classification: 47B40

Keywords and phrases: matrix, spectral operators, the resolution of the identity, spectrum

The aim of this work is to investigate the spectralicity of some matrix operator $\widetilde{A} = (A_{ij})_{n\times n}$ in the Banach space $X^n = X \times X \times \cdots \times X$, where X is a Banach space and $A_{ij} : X \to X$ are linear bounded operators.

We prove the following theorems.

Theorem 1 *Let $A_{ij} (i \neq j)$ be commutative quasinilpotent operators, A_{ii} be spectral operators and $A_{ii}A_{ij} = A_{ij}A_{ii}$. If $E_{ij}(\cdot)$ is a resolution of the identity of the operator A_{ij} then $\widetilde{A}$ is a spectral operator with a resolution of the identity $\widetilde{E}(\cdot) = (E_{ij}(\cdot))_{n\times n}$ and spectrum*

$$\sigma(\widetilde{A}) = \bigcup_{i=1}^{n} \sigma(A_{ii}).$$

Theorem 2 *If $A_{i+1,i} = I$, $i = 1, 2, \ldots, n-1$, A_{1n} is an invertible bounded operator and $A_{ij} = 0$ for other i and j, then $\widetilde{A}$ is a spectral operator with the resolution of the identity $\widetilde{E}(\cdot) = (\delta_{ij}E(\cdot))_{n\times n}$ if and only if A_{1n} is a spectral operator with the resolution of the identity $E(\cdot)$. In addition,*

$$\sigma_p(\widetilde{A}) = \bigcup\{\lambda : \lambda^n \in \sigma_p(A_{1n})\}, \quad \sigma_r(\widetilde{A}) = \bigcup\{\lambda : \lambda^n \in \sigma_r(A_{1n})\},$$

$$\sigma_c(\widetilde{A}) = \bigcup\{\lambda : \lambda^n \in \sigma_c(A_{1n})\},$$

where σ_p, σ_r and σ_c are the point, residual and continuous parts of the spectrum σ.

Note that these results are obtained with my student M.I.Ismailov and generalize and make more precise the results of [1] and [2].

References

[1] D.E. Allakhverdiev and A.M. Akhmedov, *Some classes of generalized spectral operators and their applications,* Math. USSR-Sb. 67 (1990), no. 1, 43–63.

[2] M.I. Ismailov, *On spectral representation and operator calculation of a class of operators in Banach space,* Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. 26 (2006), no. 7, 97–106.

Uniform convergence and summability of Fourier series

When, Where

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Primary AMS Classification: 42A24

Keywords and phrases: Uniform Convergence,Summability

Let (α_n) be a sequence of real numbers, where $\alpha_n > -1$, $n = 1, 2, \dots$. Suppose

$$\sigma_n^{\alpha_n}(x, f) = \sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n - 1} S_\nu(x, f) / A_n^{\alpha_n}$$

where $A_k^{\alpha_n} = (\alpha_n + 1)(\alpha_n + 2)...(\alpha_n + k)/k!$ and $S_\nu(x, f)$ is the partial sums of the trigonometric Fourier series. If (α_n) is a constant sequence then $\sigma_n^{\alpha_n}(x, f)$ coincides with the usual Cesro means.

In 1953 Nash introduced the notion of the class $\overline{\Phi}$.

Deffinition. Let Φ be a positive sequence ($\lim_{n\to\infty} \Phi(n) = +\infty$) and $\overline{\Phi}$ be the class of $2\pi-$periodic continuous function for which

$$\left| \int_a^b f(x+t) \cos nt dt \right| \leq 1/\Phi(n)$$

uniformly in x and a, b $(|b - a| \leq 2\pi)$, $n = 1, 2, \dots$.

Improving the corresponding Sat's theorem the estimation of the $||\sigma_n^{\alpha_n}(\cdot, f) - f(\cdot)||_C$ deviation in terms of the class $\overline{\Phi}$ and modulus of continuity of f, is given.

Schauder bases in Fréchet spaces and vector measures

When, Where

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Angela Albanese (speaker), *University of Salento, Italy*

Primary AMS Classification: 46A35

Keywords and phrases: Schauder basis, Fréchet spaces, Reflexive Fréchet spaces

The behavior of bases and basic sequences in reflexive Banach spaces has been completely characterized. Recall that R.C. James proved that a Banach space with a Schauder basis is reflexive if and only if the basis is both shrinking and boundedly complete, [2]. M. Zippin showed, for a Banach space X with a basis, that if every basis is boundedly complete or if every basis is shrinking, then X is reflexive, [5]. James's characterization of reflexivity was generalized to lcHs' by Dubinsky and Retherford [1], [4]. Furthermore, Kalton extended both Singer's and Zippin's results to cover lcHs' more general than Banach spaces, [3]. In particular, he proved that a sequentially complete lcHs with a Schauder basis in which every basic sequence is boundedly complete or every basic sequence is shrinking is necessarily semi-reflexive. He also showed that a complete barrelled lcHs with a normalized Schauder basis in which every normalized basis is boundedly complete, or in which every normalized basis is shrinking, is reflexive. In particular, Kalton raised the question of whether the last result remains true without the restriction of normalization on the basis, [p.265,3]. By using ideas of [3] and [5], we provide a positive answer to this question.

References

[1] E. Dubinsky, J.R. Retherford, *Schauder basis and Köthe sequence spaces,* Trans. Amer. Math. Soc. 130 (1968), 265–280.

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[5] M. Zippin, *A remark on bases and reflexivity in Banach spaces,* Israel J. Math. (1968), 74–79.

Approximation by positive operators of the Co-semigroups associated with one-dimensional diffusion equations

When, Where

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Rachida Amiar (speaker), *University of Annaba, Algeria*

Primary AMS Classification: 41A36

Keywords and phrases: Positive approximation process, Degenerate diffrential operators, Diffusion equation, Positive C_0-semigroup, Weighted function space

In a first part, we are mainly interested in studying one-dimensional second-order differential operators of the form

$$L(u) = \alpha u" + \beta u' + \gamma$$

defined on a suitable domain $D(L)$ of some weighted space of continuous functions E on a fixed arbitrary real interval, endowed with th weighted norm and the natural order. We assume that the operator $(L, D(L))$ is the generator of a C_0-semigroup $S(t)_{t\geqslant 0}$ on E and our main problem consists in constructing a suitable sequence $(M_n)_{n\geqslant 1}$ of positive operators on E (in fact, a positive approximation process on E) such that for every $t \geqslant 0$ and $f \in E$

$$S(t) = \lim_{n\to\infty} M_n^{k(n)}$$

where $(k(n))_{n\geqslant 1}$ is a sequence of positive integers such that and denotes the iterates of order. Such a representation formula can be used to derive both numerical and qualitative infomations on the semigroup and hence on the solutions of the diffusion equations associated with. In the second part, we state some general conditions which guarantee that the C_0-semigroup generated by special classes of one-dimensional second -order differential operators acting on weighted spaces of continuous functions on an arbitrary real interval can be represented as limits of iterayes of the positive linear operators constructively defined according to the method developed in the first part.

References

[1] F.Altomare, *Approximation theory methods for the study of diffusion equations. In Approximation Theory, Proceedings IDoMat' 95 (W. Muller, M. Felten, and D. H. Mache, eds.),* Mathematical Research, Vol. 86, pp. 9-26. A kademie Verlag, Berlin.

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Partial inner product spaces and operators on them

When, Where

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Primary AMS Classification: 46C50

Keywords and phrases: partial inner products, Banach scales, distribution spaces

Many families of function spaces play a central role in analysis, in particular in signal processing (e.g. wavelet or Gabor analysis). Such are L^p spaces, Besov spaces, amalgam spaces or modulation spaces. In all such cases, the parameter indexing the family measures the behavior (regularity, decay properties) of particular functions or operators. In this context, it is often said that such families should be taken as a whole and operators, bases, frames on them should be defined globally, for the whole family, instead of individual spaces.

It turns out that all these space families are scales or lattices of Banach spaces, and as such they are special cases of *partial inner product spaces (PIP-spaces)*. These objects, which may often be seen as an alternative to the theory of tempered distributions, have been studied systematically in many papers and are now the subject of an upcoming monograph [1]. The interesting fact is precisely that they allow a global definition of operators, and various operator classes on them have been defined. In this talk, we shall give an overview of PIP-spaces and operators on them, illustrating the results by space families of interest in mathematical physics and signal analysis.

References

[1] J-P. Antoine and C. Trapani, Partial Inner Product Spaces, Theory and Applications, monograph in preparation, 2009.

Some approaches to the p-Laplace operator

When, Where

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Primary AMS Classification: 47H30

Keywords and phrases: p-Laplace operator, monotone operators, nonlinear spectral theory, numerical ranges with gauge function, nonlinear resolvents

A large variety of problems in mathematics, physics, and mechanics leads to equations involving the so-called p-Laplace operator defined by

$$\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u) \qquad (1 < p < \infty).$$

Theoretically, this (nonlinear) operator has many nice properties in common with the classical (linear) Laplace operator $\Delta = \Delta_2$, but explicit calculations are in general very messy. This emphasizes the need of having information on the existence and qualitative behaviour of hypothetical solutions without knowing them.

The purpose of this talk is to illustrate how to obtain information on eigenvalue problems for the p-Laplace operator by means of four different methods. The first method is Minty's celebrated monotonicity theorem, the second one uses nonlinear spectral theory, the third one is based on so-called numerical ranges with gauge function, and the fourth one builds on nonlinear semigroup theory.

The presentation will be elementary throughout and does not require a detailed background in nonlinear analysis or PDEs.

Subnormal operators and positive definite functions

When, Where

Dragu Atanasiu, *University of Borås, Sweden*

Primary AMS Classification: 47B15

Keywords and phrases: subnormal operator ,positive definite function,completely positive definite function,dilation

In this talk we characterize the subnormal operators using the positive definite functions on the semigroup $\mathbb{N}$.We also give a new proof for the characterization of a subnormal operator from [1].

References

[1] M.Embry , *A generalization of the Halmos-Bram criterion for subnormality,* Acta Sci.Math.(Szeged) 35 (1973), 61-64.

Analytic construction of an example of orthogonal polynomials of class one

When, Where

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Mohamed J. Atia (speaker), *Gabes University, Tunisia*

Primary AMS Classification: 33C45, 42C05

Keywords and phrases: Orthogonal Polynomials, orthogonality relation, Rodrigues Formula, Hypergeometric functions.

Using **only** the following orthogonality relation

$$\int_{-1}^{1} x^{2q+1}(1-x^2)^{\alpha}(1-x)P_n^{\alpha,q}(x)P_m^{\alpha,q}(x)dx = k_n\delta_{n,m};\ n,m \geq 0,\ q \in \mathbb{N},\ \alpha > -1,$$

we give:

- The differential equation that each $\{P_n^{\alpha,q}\}_{n\geq 0}$ fulfils and we write these polynomials with hypergeometric functions.
- The recurrence coefficients.
- The Rodriguez formula for these polynomials.
- An electrostatic application for $\{P_{2n}^{\alpha,q}\}_{n\geq 0}$.
- Some informations about the Zeros.

References

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[2] M. J. Atia, *Explicit representations of some orthogonal polynomials,* Integral Transforms and Special Functions Vol. 18, No. 10, October (2007), 731-742.

[3] W. Van Assche, *The impact of stieltjes work on continued fractions and orthogonal polynomials,* arXiv:math/9307220v1 [math.CA] 9 Jul 1993. In: G. van Dijk, Editor, Thomas Jan Stieltjes Oeuvres Compltes/Collected Papers, Vol. I, Springer, Berlin (1993), 5–37.

Multifractality, local p-convexity and diametral dimensions

When, Where

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Primary AMS Classification: 46A16

Keywords and phrases: Multifractal analysis, pointwise Hölder regularity, p-convexity, diametral dimensions

Multifractal analysis led to the consideration of a family of new function spaces called S^ν, defined by asymptotics of wavelet coefficients, which generalize countable intersections of Besov spaces. They are separable, complete metrizable linear spaces with interesting topological properties. For instance, their local convexity index p_0 is related to certain features of the maximal/prevalent spectrum of singularities. They are Schwartz spaces, even algebras under certain conditions, but when $p_0 < 1$ i.e. the spaces are not locally convex, the study of nuclearity does not make sense and has to be replaced by the computation of their diametral dimension.

Generalized Fourier-Stieltjes algebras on a locally compact group

When, Where

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Primary AMS Classification: 46L07

Keywords and phrases: Operator spaces, Fourier-Stieltjes algebras, unitary representation, Completely contractive Banach algebra

Let G be a locally compact group and $\mathbf{H}$ be a Hilbert space. Weakly continuous and completely positive definite maps [1] on a locally compact group are introduced as an extension of positive definite maps. We denote by $CB(G, \mathbf{B}(\mathbf{H}))$ the linear span of all weakly continuous and completely positive definite maps $\phi : G \to \mathbf{B}(\mathbf{H})$. If $\mathbf{H} = \mathbb{C}$ (complex numbers) then $CB(G, B(\mathbb{C}))$ is just the Fourier-Stieltjes algebra $B(G)$[2]. In this paper we study $CB(G, \mathbf{B}(\mathbf{H}))$ as an extension of $B(G)$ from operator space theory point view.

We impose a dual operator space structure on $CB(G, \mathbf{B}(\mathbf{H}))$ and then by using a well-known generalization of Stinespring's representation theorem to extend and define the point wise product on $CB(G, \mathbf{B}(\mathbf{H}))$ which make it a completely contractive commutative Banach algebra. We also show there is a dual operator spaces identification between $CB(G, \mathbf{B}(\mathbf{H}))$ and the normal spatial tensor product $B(G)\overline{\otimes}\mathbf{B}(\mathbf{H})$. Then we calculate product on $B(G)\overline{\otimes}\mathbf{B}(\mathbf{H})$ which is imposed by $CB(G, \mathbf{B}(\mathbf{H}))$. In fact we show the imposed product on $B(G)\overline{\otimes}\mathbf{B}(\mathbf{H})$ is an extension of the following multiplication on decomposable tensors

$$\phi \otimes v \bullet \psi \otimes w = \phi\psi \otimes v * w$$

where $v * w$ is the schur product of operators v and w.

[1] V.I Pualsen, *Completely Bounded Maps and Operator Algebras,* Cambridge University Press, 2002.

[2] P. Eymard, *L'algèbre de Fourier d'un group localement compact,* Bull. Soc. Math. Franc (1964), 181-236.

Dissipative Sturm-Liouville Operators

When, Where

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Primary AMS Classification: 34B24

Keywords and phrases: Differantial operator, Spectral theory

In this talk, using Livsic's Theorem [1], we shall discuss the problem of completeness of the system of eigenfunctions and associated functions of dissipative operators generated by the Sturm-Liouville differantial expression on the semi-axis in Weyl's limit-circle case ([2], [3]).

References

[1] I. C. Gokhberg and M. G. Krein, *Introduction to the Theory of Linear Nonselfadjoint Operators,* Am. Math. Soc. Providence (1969).

[2] A. M. Krall and A. Zettl, *Singular self-adjoint Sturm-Liouville problems,* J. Differential Integral Equations 1 (1988), 423-432.

[3] M. A. Naimark, *Linear Differential Operators II,* Ungar, New York (1968).

Some New Fixed Point Theorems in Modular Spaces

When, Where

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Primary AMS Classification:

Keywords and phrases: Modular space; Fatou property; Contractive self-mapping; Asymptotically regular mapping; $\triangle_2-$ condition; Gauge function

Prinov in [1] proved that a continuous assymptotically regular mapping T on a complete metric space satisfying certain conditions, has a contractive fixed point. We generalize this theorem in a complete modular space (X_ρ, ρ). The conditions contain the existence of a gauge function φ such that for $0 < l < c$:

$$\rho(c(Tx - Ty)) \leq \varphi\varrho(l(x - y)) \tag{1}$$

for all $x, y \in X_\rho$; and

$$\rho(c(Tx - Ty)) < \varrho(l(x - y)), \tag{2}$$

where ϱ is defined.

References

[1] Petko D. Prinov, *Fixed point theorems in metric spaces.*, Nonlinear Analysis., Volume 64 (2006), 546-557.

On boundedness of maximal operators with respect to Vilenkin systems

When, Where

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Primary AMS Classification: 42C10

Keywords and phrases: Walsh-Paley system, Vilenkin groups, Fejér and Marczinkiewicz means of Fourier series, Hardy spaces

In this lecture we talk about the boundedness of maximal operator $\sigma^* = \sup_n |\sigma_n|$ on Vilenkin- and double Vilenkin-systems with Fejér and with the Marcinkiewicz-Fejér means. We observe similar properties on the d-dimensional Walsh-Paley system.

This presentation is a survey of published and submitted papers listed below.

References

[1] Blahota, I., Gát, G., Goginava, U., *Maximal operators of Fejér means of Vilenkin-Fourier series,* Journal of Inequalities in Pure and Applied Mathematics (JIPAM) 7 (4) (2008).

[2] Blahota, I., Gát, G., Goginava, U., *Maximal operators of Fejér means of double Vilenkin Fourier series,* Colloq. Math. 107 no. 2 (2007), 287-296.

[3] Blahota, I., Goginava, U., *The Maximal Operator of the Marcinkiewicz-Fejr Means of the 2-Dimensional Vilenkin-Fourier Series,* Studia Scientiarum Mathematicarum Hungarica 45 (3) (2008).

[4] Blahota, I., Goginava, U., *The martingale Hardy type inequality for the maximal operator of the (C, α) means of cubic partial sums of the d-dimensional Walsh-Fourier series,* (submitted).

The role of Conical Measures in Functional Analysis

When, Where

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Primary AMS Classification: 46A40

Keywords and phrases: Convexity, Ordered vector spaces

G. Choquet passed away 3 years ago. He was my adviser. He has proved a celebrated Theorem of *Integral Representation*, in the framework of metrizable convex compact sets, contained in a Hausdorff locally convex space. To cope with the case of a convex cone *without* compact base, he introduced the following notion of *conical measure* :

By definition, a conical measure, on a Hausdorff locally convex space E, with continuous dual E', is a *positive linear form* on the *vector lattice* of functions on E generated by E'.

In this communication, I intend to demonstrate how conical measures are also quite useful in various other fields, namely:

1) To study *Zonoforms*, (E. Bolker, G. Choquet, see [1]).
2) In the theory of *Statistical Decision*, (L. Le Cam, see [1]).
3) In the theory of *Vector Measures*, (I. Kluvanek, see [1]).
4) To study *Ordered Banach spaces*, (see [2]).
5) To study the *Dedekind completion* of an ordered vector space, (see [3]).

References

[1] R. Becker , *Convex cones in Analysis,* Hermann, Paris, (2006).

[2] R. Becker, *Ordered Banach spaces,* Hermann, Paris, (2008).

[3] R. Becker, *Vector lattices associated with ordered vector spaces,* Mediterr. J. M. (2010).

Absolutely Convergent Extensions of Nonclosable Positive Linear Functionals

When, Where

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Primary AMS Classification: 46H05

Keywords and phrases: Absolutely convergent extension, closable sesquiliner form

If $\mathfrak{A}[\tau]$ is a topological *-algebra, with separately continuous multiplication and continuous involution *, and $\mathfrak{A}_0$ is a dense *-subalgebra of $\mathfrak{A}$ where a positive linear

functional ω (i.e. $\omega(a^*a) \geq 0$, for every $a \in \mathfrak{A}_0$) is defined, one may look for an extension of ω to some subspace or subalgebra of $\mathfrak{A}$. This problem has easy solutions if, e.g. ω is τ-continuous or if it closable i.e. when $\omega(a_n)$ is forced to tend to 0 whenever the sequence $\{a_n\}$ tends to 0 and $\{\omega(a_n)\}$ converges.
Provided that certain *regularity* assumptions on both the *-algebra $\mathfrak{A}$ and on the functional ω are satisfied (the crucial one consists in assuming that any hermitian element $x \in \mathfrak{A}_0$ decomposes uniquely as $x = x_+ - x_-$, where x_+, x_- are positive elements in $\mathfrak{A}_0$, whose product vanishes) we construct an *absolutely convergent slight extension* of ω defined on a $*$-invariant subspace of $\mathfrak{A}$. An extension $\hat{\omega}$ of ω is said to be *slight* if $\hat{\omega}$ is the linear functional having as graph a given subspace G of $\overline{G_\omega}$ (the closure of the graph G_ω of ω) with the property that G does not contain couples $(0, \ell)$ with $\ell \in \mathbb{C}$, $\ell \neq 0$. An extension $\hat{\omega}$ of ω is called *absolutely convergent* when $x = x^* \in \mathcal{D}(\hat{\omega}) \Leftrightarrow x_+, x_- \in \mathcal{D}(\hat{\omega})$, with $\mathcal{D}(\hat{\omega})$ the domain of $\hat{\omega}$.
Besides, since continuity and closability of ω are rather strong hypotheses (the latter is rarely realized in practice), we are interested to situations where ω is nonclosable, but the positive sesquilinear form φ_ω on $\mathfrak{A}_0 \times \mathfrak{A}_0$, defined by $\varphi_\omega(a, b) := \omega(b^*a)$, $a, b \in \mathfrak{A}_0$ is *closable* i.e. when $a_n \xrightarrow{\tau} 0$ and $\varphi_\omega(a_n - a_m, a_n - a_m) \to 0$ as $n, m \to \infty \Rightarrow \varphi_\omega(a_n, a_n) \to 0$. If ω is an admissible (i.e. for every $b \in \mathfrak{A}_0$ there exists $\gamma_b > 0$ such that $|\omega(a^*ba)| \leq \gamma_b \omega(a^*a)$, $\forall a \in \mathfrak{A}_0$) trace with closable associated positive sesquilinear form, then ω has an absolutely convergent extension.

Multivariate Bernstein-Durrmeyer Operators with Arbitrary Weight Functions

When, Where

Elena E. Berdysheva (speaker), *University of Hohenheim, Germany*

Kurt Jetter, *University of Hohenheim, Germany*

Primary AMS Classification: 41A36

Keywords and phrases: Bernstein basis polynomials, Bernstein-Durrmeyer operator, reproducing kernel Hilbert space

We introduce a class of Bernstein-Durrmeyer operators with respect to an arbitrary measure on a multi-dimensional simplex, and a class of more general polynomial integral operators with a kernel function constructed as a weighted sum of the

Bernstein basis polynomials. These operators generalize the well-known Bernstein-Durrmeyer operators with Jacobi weights. We investigate properties of the new operators. In particular, we study the associated reproducing kernel Hilbert space and show that the Bernstein basis functions are orthogonal in the corresponding scalar product. We discuss spectral properties of the operators. We make first steps in understanding convergence of the operators. In particular, we describe a class of weights on the simplex, for which the uniform convergence holds — these weights can be estimated from above and from below by two different Jacobi weights.

Eigenvalues of large Hankel matrices

When, Where

Christian Berg, *Copenhagen University*, *Denmark*

Primary AMS Classification: 15A18

Keywords and phrases: Hankel matrices, orthogonal polynomials

Let μ denote a positive measure on the real line with moments (s_n) of any order and infinite support. The corresponding Hankel matrices $H_n = (s_{i+j}), 0 \leq i, j \leq n$ of size $n + 1, n = 0, 1, \dots$ are positive definite. The smallest eigenvalue λ_n of $H_n, n = 0, 1, \dots$ form a decreasing sequence. It is known by a theorem of Chen, Ismail and the speaker that λ_n tends to zero if and only if μ is uniquely determined by the moments. We show that arbitrarily slow and fast convergence to zero of λ_n is possible. We also revisit the indeterminate case, where λ_n is bounded below by a positive constant.

The talk is based on the following joint work

References

[1] C. Berg and R. Szwarc, *The smallest eigenvalue of Hankel matrices,* Manuscript

Banach lattices which are order uniformly noncreasy

When, Where

Anna Betiuk-Pilarska (speaker), *M. Curie-Skłodowska University, Poland*

Stanisław Prus, *M. Curie-Skłodowska University, Poland*

Primary AMS Classification: 46B42

Keywords and phrases: uniform monotonicity; order uniform smoothness; order uniform noncreasiness; weak fixed point property.

One of the main problems of metric fixed point theory concerns existence of fixed points of nonexpansive mappings. In [4], Maurey applied an ultrapower technique to this problem. His approach turned out to be very fruitful. In particular it enabled many authors to prove fixed point theorems for nonexpansive mappings in Banach spaces satisfying various geometric properties (see [1] and [2]). One of them was introduced in [5] as a combination of the classical geometric properties: uniform convexity and uniform smoothness. Banach spaces with this property are called uniformly noncreasy. We present a property which is related to order. Banach lattices with this property will be called order uniformly noncreasy. Our definition was inspired by Kurc [3] who considered uniform monotonicity and order uniform smoothness of Banach lattices and showed a duality theorem for these properties. The property can be seen as a combination of uniform monotonicity and order uniform smoothness and it is essentially weaker than both of them.

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An extension of Haar's Theorem

When, Where

Aldric Brown, *University College London, UK*

Primary AMS Classification: 41A50

Keywords and phrases: continuous function, metric projection, lower semi-continuity

Let T be a (locally) compact Hausdorff space and $C(T)$ the space of real continuous functions on T (which vanish at infinity), equipped with the uniform norm. Let G be a linear subspace of $C(T)$ of finite dimension n. Then P_G dnotes the set-valued metric projection of $C(T)$ onto G; that is, $P_G(f)$ is the non-empty closed convex set of best approximations to f from G, for each f in $C(T)$.

Wu Li (1989, see also Brown 2005) charactereised those G for which the metric projection is lower semi-continuous. A consequence of the characterisation is that for such a G the restriction of G to a component of T is Chebyshev. If no component of T is a single point then the characterisation is equivalent to one whih generalises the familiar determinantal form of Haar's characterisation of Chebyshev subspaces G of $C(T)$. The latter is amenable to calculation and in the case

$$T = [0, 1] \times \{1, \dots, k\}$$

it is possible to construct many non-Chebyshev subspacs of $C(T)$ which demonstrably have metric projections which are lower semi-continuous.

On the generalized Jacobi-Stirling numbers

When, Where

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Gradimir V. Milovanović, *Nis, Serbia*

Primary AMS Classification: 33C65

Keywords and phrases: Jacobi polynomials, Stirling numbers of the second kind, Jacobi-Stirling numbers

The Jacobi-Stirling numbers of the first and second kinds were introduced in 2006 by Littlejohn and oth. In this paper, we introduce the concept of the generalized Jacobi-Stirling numbers of the first and second kinds. We investigate several interesting properties and relationships involving the classical as well as the generalized Stirling numbers and polynomials. As an application, we will treat the numbers definied by Letterio Toscano.

On a generalization of Kantorovich operators on simplices and hypercubes

When, Where

Francesco Altomare, *University of Bari, Italy*

Mirella Cappelletti Montano (speaker), *University of Bari, Italy*

Vita Leonessa, *University of Basilicata, Italy*

Primary AMS Classification: 41A36.

Keywords and phrases: Positive approximation process. Rate of convergence. N-dimensional hypercube. N-dimensional simplex.

In this talk, we shall discuss some results contained in [2], where we introduce and study two new sequences of positive linear operators acting, respectively, on the space of all Lebesgue integrable functions on the N-dimensional hypercube and on the space of all Lebesgue integrable functions on the N-dimensional simplex, $N \geq 1$.

In particular, we prove that these sequences are approximation processes with respect both to the sup-norm and to the L^p-norm and we present several estimates of their rate of convergence by means of suitable moduli of smoothness.

These operators are a natural generalization to these multi-dimensional settings of a certain sequence studied in [1] and, in particular, of Kantorovich operators on $[0, 1]$.

References

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[2] F. Altomare, M. Cappelletti Montano and V. Leonessa, *On a generalization of Kantorovitch operators on simplices and hypercubes,* preprint, 2009.

Approximation of finitely additive functions valued into topological groups

When, Where

Paola Cavaliere (speaker), *Università di Salerno*, *Italy*

Paolo de Lucia, *Università di Napoli "Federico II"*, *Italy*

Hans Weber, *Università di Udine*, *Italy*

Primary AMS Classification: 28A10.

Keywords and phrases: Additive set functions. Approximation results.

We present conditions ensuring that any finitely additive function, which is defined on an atomless Boolean algebra and takes values into a Hausdorff topological commutative group $\mathcal{G}$, is the pointwise limit of exhaustive and strongly continuous functions. In particular, our results extend those of [1] and [2], where $\mathcal{G}$ is assumed to be $\mathbb{R}$ and a Banach space, respectively.

References

[1] K. P. S. Bhaskara Rao - M. Bhaskara Rao, *Charges on Boolean algebras and almost discrete spaces,* Mathematika 20 (1973), 214–223.

[2] V. M. Klimkin & M. G. Svistula, *On the pointwise limit of vector charges with the Saks property,* Math. Notes 74 (2003), 385–392.

Szeő asymptotics

When, Where

Jacob S. Christiansen, *University of Copenhagen, Denmark*

Primary AMS Classification: 42C05

Keywords and phrases: Orthogonal polynomials, Isospectral torus

In the talk, I'll discuss Szegő asymptotics for orthogonal polynomials on several intervals. In particular, the role of the Szegő condition will be discussed.

Quadrature rules for high-oscillatory and periodic functions and some applications to integral equations

When, Where

Maria Carmela De Bonis, *University of Basilicata, Italy*

Primary AMS Classification: 65D32

Keywords and phrases: Quadrature rules, orthogonal polynomials, integral equations

The numerical evaluation of integrals of the following type

$$\int_a^b f(x)g(\omega x)\ dx, \quad -\infty \leq a < b \leq +\infty, \tag{1}$$

where f is a Rieman integrable function on $[a,b]$ and $g(\omega x)$ is a highly oscillatory function, interested several authors and actually is a special chapter of the numerical integration. The difficulties that appear in the evaluation of (1) are both theoretical and numerical. In fact, if, for example, we apply a $m-$points Gauss-Legendre quadrature rule to the integral

$$\int_0^1 e^{2x}\sin(5000x)\ dx, \tag{2}$$

the remainder term satisfies the following estimate

$$|R_m(f)| = \emptyset\left[\left(\frac{3399}{m}\right)^{2m}\right].$$

Therefore, in order to evaluate (2) with some exact digits we need to use a Gauss-Legendre quadrature rule based on a number of points m greater than 3399. But this is not realistic.

In this talk, we will show that, in several cases, a dilation of the integration interval results simple and efficient. Moreover we will consider applications of such a procedure for approximating the solutions of some integral equations.

Role of noncompatible and discontinuous mappings to prove coincidence and common fixed point theorems in various spaces

When, Where

Bhavana Deshpande, *Govt. Arts and Science P.G. College Ratlam, India*

Primary AMS Classification: 47H10, 54H25

Keywords and phrases: Noncompatible maps, Fuzzy metric spaces, Menger spaces, Intuitionistic fuzzy metric spaces, Intuitionistic Menger space, Common fixed point

In this talk, we will discuss single valued and hybrid pair of compatible mappings. As applications of noncompatible mappings we will discuss some coincidence and fixed point theorems in metric spaces, fuzzy metric spaces, Menger spaces and intuitionistic fuzzy metric spaces etc. We will point out that continuity of any mpping is not necessary for the existence of common fixed point for noncompatible mappings. We will give some examples to validate our results. We will discuss some recent results and applications.

On rank one perturbation of continuous spectrum which generates prescribed finite point spectrum

When, Where

Evegny Cheremnikh, *Lviv Polytechnic National University*, *Ukraine*

Fatma Diaba (speaker), *University of Annaba*, *Algeria*

Primary AMS Classification: 17A20, 17A30

Keywords and phrases: finite point spectrum, perturbation of continuous spectrum, Hilbert transformation, set of zeros

The perturbations of Nevanlinna type functions which preserve the set of zeros of this function or add to this set a new points are discussed.

References

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On Feller semigroups associated with modified Bernstein-Schnabl operators

When, Where

Francesco Altomare, *University of Bari, Italy*

Mirella Cappelletti Montano, *University of Bari, Italy*

Sabrina Diomede (speaker), *University of Bari, Italy*

Primary AMS Classification: (47D06)

Keywords and phrases: Modified Bernstein-Schnabl operator, degenerate second-order elliptic differential operator, positive operator semigroup.

We study second-order elliptic differential operators of the form

$$V_T(u)(x) := \sum_{i,j=1}^{p} \alpha_{ij}(x)\frac{\partial^2 u}{\partial x_i x_j}(x) + \sum_{i=1}^{p} \beta_i(x)\frac{\partial u}{\partial x_i}(x) + \gamma(x)u(x), \quad (x \in K)$$

with continuous coefficients defined on finite dimensional convex compact sets. In the spirit of the pioneer paper [1], we show that the closure A of V_T generates a Feller semigroup which may be approximated by a suitable sequence of positive linear operators. Hence, we obtain an explicit representation of the solution to the abstract Cauchy problem related to A.

References

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Recent Results on Generalized Inverses

When, Where

Dragan Djordjevic, *Faculty of Sciences and Mathematics, University of Nis, Serbia*

Primary AMS Classification: 47A05

Keywords and phrases: Generalized inverses, Reverse order law, Perturbations, Operator equations

We shall present some new results concerning the generalized inverses of linear bounded operators. The results concern the reverse order law, additive formulae and some operator equations.

Uncertainty principles on compact Riemannian manifolds

When, Where

Wolfgang Erb, *Helmholtz Center Munich, Germany*

Primary AMS Classification: 26D10

Keywords and phrases: Uncertainty principles, Riemannian manifolds, Dunkl operators

Based on a result of Rösler and Voit for ultraspherical polynomials, we derive an uncertainty principle for compact Riemannian manifolds M. Similar as for the classical Heisenberg principle or the Breitenberger principle on the unit circle, the proof of the uncertainty rests upon operators in Hilbert spaces. As a frequency operator, we will construct a special differential-difference operator, a so called Dunkl operator, which plays the role of a generalized root of the radial part of the Laplace-Beltrami operator on M. Subsequently, we will show with a family of Gaussian-like functions that the deduced uncertainty inequality is in fact asymptotically sharp. Finally, we specify in more detail the uncertainty principles for well known Riemannian manifolds like the d-dimensional unit sphere and the projective spaces.

Zeros of polynomials with restricted coefficients

When, Where

Tamas Erdelyi, *Texas A&M University, Texas, USA*

Primary AMS Classification: 11B75

Keywords and phrases: polynomials, restricted coefficients, location of zeros

We focus on estimating the number of distinct sign changes a polynomial with coefficients in -1,0,1 may have in (0,1). We improve and extend an old result of Bloch and Polya (1932) in this direction. Various other questions about the (real) zeros of polynomials with restricted coefficients, such as Littlewood polynomials, may be discussed.

On the approximate convexity and submonotonicity in locally convex spaces

When, Where

Ali Farajzadeh, *Islamic Azad University- Kermanshah Branch , Iran*

Primary AMS Classification: 47A15

Keywords and phrases: Locally Lipschitz mapping; Approximate convexity; Submonotonicity; Locally convex space.

In this talk, we introduce some new concepts of locally Lipschitz mappings, Clarke subdifferential, approximate convexity and submonotonocity in locally convex spaces. We show that, if f is approximately convex and bounded above, then f is locally Lipschitz. We also prove that a Lipschitz function is approximately convex if and only if its Clarke subdifferential is a submonotone operator.

References

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The Shepard operator on the real semiaxis

When, Where

Luisa Fermo, *Università degli Studi della Basilicata, Italy*

Primary AMS Classification: 41A36

Keywords and phrases: Shepard operator

In this talk we approximate functions defined on $(0,\infty)$ by a positive linear operator based on the zeros of the Laguerre polynomials. To be more precise, we consider the following Shepard-type operator

$$\mathcal{S}_m^{(\alpha)}(f,x) = \frac{\displaystyle\sum_{k=1}^{j}(x-x_k)^{-2}f(x_k)}{\displaystyle\sum_{k=1}^{j}(x-x_k)^{-2}},$$

where $x_1 < x_2 < ... < x_m$ are the zeros of the polynomial $p_m(w_\alpha)$ which is orthonormal to the weight $w_\alpha(x) = x^\alpha e^{-x}$, $\alpha > -1$ and $x_j = \min_{1\le k\le m}\{x_k : x_k \ge 2m\}$. A result of the talk is

$$\|[f - \mathcal{S}_m^{(\alpha)}(f)]w_\gamma\|_\infty \le C \sum_{i=1}^{j} \frac{1}{i^2}\omega_\varphi\left(f, \frac{i}{\sqrt{m}}\right)_{w_\gamma}$$

where $C \ne C(m, f)$, $w_\gamma(x) = x^\gamma e^{-x}$, $\gamma \ge 0$ and $\omega_\varphi(f, t)_{w_\gamma}$ denotes the weighted φ-modulus of smoothness.

Moreover, other operators will be also considered.

Localized kernels and approximation processes on Riemannian manifolds

When, Where

Frank Filbir (speaker), *Helmholtz Center Munich, Germany*

Hrushikesh N. Mhaskar, *California State University, U.S.A.*

Primary AMS Classification: 41A35

Keywords and phrases: Quadrature formulas, Riemannian manifold, diffusion polynomials

Let $\{\phi_j\}$ be an orthonormal system on a Riemannian manifold $\mathbb{X}$, $\{\ell_j\}$ be a nondecreasing sequence of numbers with $\lim_{j\to\infty} \ell_j = \infty$. A diffusion polynomial of degree L is an element of the span of $\{\phi_k \ : \ \ell_l \le L\}$. We study a kernel based approximation processes of the form

$$\sigma_L f(x) = \int_{\mathbb{X}} f(y)\, \Phi_L(x, y)\; d\mu(x) = \sum_{j=0}^{\infty} H\big(\frac{\ell_j}{L}\big)\; \langle f, \phi_j\rangle \phi_j(x), \qquad (1)$$

where the kernel $\Phi_L(x, y) = \sum_{j=0}^{\infty} H\big(\frac{\ell_j}{L}\big)\, \phi_j(x)\phi_j(y)$ is defined by a suitable function H. We will address two problems. Firstly, we show how well localized kernels lead to a good approximation process. Secondly, we consider discrete versions of (1) based on quadrature formulas exact for diffusion polynomials of certain degree which work for scattered data.

References

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[3] M. Maggioni, H. N. Mhaskar , *Diffusion polynomial frames on metric measure spaces,* Appl. Comput. Harmon. Anal. 24 (2008) 329–353.

Classes of operator valued functions, with relation to integral multipliers

When, Where

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Primary AMS Classification: 46G10

Keywords and phrases: Strongly (weakly) measurable function, integral multiplier function, p-integrable operator valued function.

In [3], the author discussed a key lemma (from the paper [2]) for strongly measurable operator valued functions and some applications thereof. Following [3], the authors in [1] considered examples of operator valued functions which are measurable with respect to the strong operator topology (and not strongly measurable) and which do or don't satisfy the result of the lemma (in [3]). This led them to introduce the *strongly μ-normable* functions. The authors in [1] then proved some results in [3] for this larger class of functions. In this talk we consider results in [1] and [2] for (classes of) functions with weaker measurability properties, in particular obtaining characterizations of spaces of operator valued multipliers acting boundedly on Banach space valued L^p-spaces (of Bochner integrable functions) and remark on some tensor product characterizations of spaces of integral multiplier functions.

References

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A Nyström method for $2s$-th order BVP

When, Where

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Primary AMS Classification: 65R20

Keywords and phrases: Boundary value problem, Fredholm integral equation, Nyström method

In this talk we illustrate a numerical method to approximate the solutions of the following boundary value problems of order $2s$

$$\left\{\begin{array}{l} f^{(2s)}(x) + a(x)f(x) = g(x) \\ f^{(i)}(\pm 1) = 0, \quad i = 0, \ldots, s-1 \\ \text{ or} \\ f^{(2i)}(\pm 1) = 0, \quad i = 0, \ldots, s-1 \end{array}\right. , \tag{1}$$

where g and a are known functions.
We reduce problems (1) to the equivalent Fredholm integral equations. Then we apply a Nyström method to solve the obtained integral equations. We prove that the procedure is stable and convergent. We show some numerical tests confirming our theoretical results.

The Metric Theory of Functional Solutions of Conservation Laws Systems

When, Where

Valeriy Galkin, *Obninsk State University, Russia*

Primary AMS Classification: 35L65

Keywords and phrases: Cauchy problem, functional solutions theory, convergence of approximate methods

We consider the correctness problem and convergence of approximate methods for nonlinear systems of conservation laws (Cauchy problem)

$$\partial_t u^{(\omega)}(x,t) + \sum_{j=1}^{n} \partial_{x_j} f_j^{(\omega)}(u,x,t) = S^{(\omega)}(u,x,t), \tag{1}$$

$$x \in \mathbb{R}_n, \quad t > 0, \quad \omega \in \Omega, \quad u|_{t=0} = u_0,$$

where $u = \{u^{(\omega)}\}$ is unknown vector-function, $x \in \mathbb{R}_n$ are space coordinates, t is the time, Ω are parameters, numbering equations (the set Ω is the metric space). The concept of functional solutions for (1) in Tikhonov topology was introduced in [1]. We consider functional solutions of (1) and background of approximate methods convergence in metric spaces.

Theorem 1 *Let approximate method for Cauchy problem* (1) *is uniformely stable and this method weakly approximates* (1). *Then approximations converges to global functional solution of* (1) *in the metric space of Young functionals attached to Sobolev solutions.*

References

[1] V. A. Galkin, *Smoluchowskii Equation,* FIZMATLIT, Moscow (2001), 336 P.

Approximation properties of one and two dimensional Fejér and Marczinkiewicz means of Fourier series with respect to Walsh and Vilenkin systems

When, Where

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Primary AMS Classification: 42C10

Keywords and phrases: Vilenkin groups, Fejér and Marczinkiewicz means of Fourier series, norm and almost everywhere convergence

Let $m := (m_k, k \in \mathbb{N})$ be a sequence of integers each of them not less than 2. Let Z_{m_k} denote the discrete cyclic group of order m_k and G_m be the complete direct product of the groups $Z_{m_k} (k \in \mathbb{N})$. The characters of G_m, $\psi := (\psi_n : n \in \mathbb{N})$ is called a Vilenkin system. If $m_j = 2$ for all j, then we are talking about the Walsh group or Walsh system. Define the nth Fejér or Nörlund logarithmic means as follows

$$\sigma_n f := \frac{1}{n}\sum_{k=0}^{n-1} S_k f, \quad t_n f := \frac{1}{\log n}\sum_{k=0}^{n-1} \frac{S_k f}{n-k}.$$

The characters of $G_m \times G_m$ are called the two-dimensional Vilenkin functions. The two-dimensional Fejér, cubical Nörlund logarithmic and Marczinkiewicz means of the two-variable integrable function $f \in L^1(G_m \times G_m)$:

$$\sigma_{n_1,n_2} f := \frac{1}{n_1 n_2}\sum_{k_1=0}^{n_1-1}\sum_{k_2=0}^{n_2-1} S_{k_1,k_2} f, \quad t_n f := \frac{1}{\log n}\sum_{k=0}^{n-1} \frac{S_{k,k} f}{n-k}, \quad z_n f := \frac{1}{n}\sum_{k=0}^{n-1} S_{k,k} f.$$

The aim of this talk is to give a résumé of the recent developments of the convergence properties of these means. Besides, we prove that it is not possible to reconstruct a two-dimensional integrable function from its Marczinkiewicz means with respect to the L^1 norm convergence in the situation of an unbounded generating sequence m.

Converse inequalities for Meyer-König and Zeller operators of finite type

When, Where

Ioan Gavrea, *Technical University of Cluj-Napoca, Romania*

Primary AMS Classification: 41A25

Keywords and phrases: Meyer-König and Zeller operators, strong converse inequalities, modulus of smoothness

In [2], we have defined a new sequence (L_n) of Meyer-König and Zeller operators of finite type. In this note, we will give a strong converse inequality of type B in the terminology of [1] for $L_n(f)$.

References

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[2] I. Gavrea, T. Trif, *The rates of convergence by certain new Meyer-König and Zeller operators of finite type,* Rendiconti del Circolo Matematico di Palermo, Serie II, Suppl. 76 (2005), 375–394.

Maximal operators of Logarithmic means of two-dimensional Walsh-Fourier series

When, Where

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Primary AMS Classification: 42C10

Keywords and phrases: Walsh function, Hardy space, maximal operator

The rectangular partial sums of the two-dimensional Walsh-Fourier series denote by $S_{i,j}f$. The Riesz logarithmic means and the logarithmic means of cubic partial sums of two-dimensional Walsh-Fourier series are defined as follows

$$R_n f := \frac{1}{\log(n+1)} \sum_{j=1}^{n} \frac{S_{j,j}f}{j}, \; L_n f := \frac{1}{\log(n+1)} \sum_{j=0}^{n-1} \frac{S_{j,j}f}{n-j}$$

respectively.

Denote by H_p ($p > 0$) Hardy martingale space. Let

$$R^* f := \sup_n |R_n f|, \; L^* f := \sup_n |L_n f| \; .$$

We proved that the following are true:

Theorem 1 *Let $f \in H_{2/3}$. Then the maximal operator R^* is bounded from martingale Hardy space $H_{2/3}$ to the space weak-$L_{2/3}$.*

Theorem 2 *Let $0 < p \leq 2/3$. Then there exists a martingale $f \in H_p$ for which* $\|R^* f\|_p = +\infty$.

Theorem 3 *Let $0 < p \leq 1$. Then there exists a martingale $f \in H_p$ for which* $\|L^* f\|_p = +\infty$.

Stochastic processes in Riesz spaces

When, Where

Jacobus Grobler, *North-West University, Potchefstroom, South Africa*

Primary AMS Classification: 46A40

Keywords and phrases: Stochastic process, Riesz space, Stopping time, Sub-Martingale, Doob-Meyer

Riesz spaces provide a natural environment for the study of abstract stochastic processes. This has been shown in a series of papers by W.-C.Kuo, C.C.A. Labuschagne and B.A. Watson (see for instance [2] and [3]) in which the discrete case has been considered. The continuous case offers some challenges even in the concrete case (see [1]) and more so in the abstract case. We explore the continuous case in Riesz spaces and show that the Doob-Meyer decomposition can be proved. In order to do this right continuous natural and predictable increasing processes and stopping times are necessarily considered.

[1] I. Karatzas and S. Shrive, *Brownian Motion and Stochastic Calculus,* Springer Verlag, Heidelberg-New York-Berlin, 1991.

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Some Remarks on Generalized Interpolation

When, Where

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Primary AMS Classification: 41A05

Keywords and phrases: Interpolation, Divided Differences

Interpolation, by polynomials or other functions, is a rather old method in applied mathematics. A modern approach to the theory of interpolation is to connect the polynomial interpolation to the theory of polynomial ideals. We define a generalized interpolation scheme and generalize the notion of divided difference in the case of abstract interpolation. We use our generalized divided difference to obtain new Brezinski and Popoviciu-Steffensen-Leibniz type Formulas.

References

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Partial differential equations of higer order and random fields

When, Where

Niels Jacob, *Swansea, United KingdomName*

A simple but surprising observation is that families of multi-parameter probability measures are related to higher order partial differential operators, or more generally pseudo-differential operators. However these operators do in general not fit into standard classes. We will first explain these relations in detail. Then we will turn to the harder problem : Given a certain higher order (peudo-)differential operator. When is it possible to associate with such an operator a multi-parameter family of probability measures and what type of stochastic processes can we associate with such a family of probability measures. Closely realted is an analytic counter part, namely the generation of multi-parameter families of operators being related to the given (pseudo-)differential operators as is a one-parameter semi-group of operators to its generator.

Convolution product associated with the spherical mean operator on distribution spaces with exponential growth

When, Where

Mourad Jelassi, *Al Jouf University, Kingdom of Saudi Arabia*

Primary AMS Classification: 42B35

Keywords and phrases: Covolution, spherical mean operator, distribution spaces with exponential growth

Using the harmonic analysis associated with the spherical mean operator we study the Fourier transform and convolution product on spaces of distribution spaces with exponential growth.

On Mapping Properties of Integral Operators with Logarithmic Kernels

When, Where

Peter Junghanns (speaker), *Chemnitz University of Technology, Germany*

Giovanni Monegato, *Politecnico di Torino, Italy*

Primary AMS Classification: 45B05

Keywords and phrases: Fredholm integral equations of first kind, weakly singular kernels, Sobolev-like spaces

For integral operators $\mathcal{K}_\kappa$ defined by

$$(\mathcal{K}_\kappa u)(x) = -\frac{1}{\pi}\int_{-1}^{1}(y-x)^\kappa \ln|y-x|\,u(y)\,dy\,,\quad -1<x<1\,,$$

$\kappa = 0, 1, 2$, we present mapping properties in scales of weighted Sobolev-like spaces and give sufficient conditions for the existence of integrable solutions of the respective Fredholm integral equations of first kind

$$-\frac{1}{\pi}\int_{-1}^{1}(y-x)^\kappa \ln|y-x|\,u(y)\,dy = f(x)\,,\quad -1<x<1\,.$$

Moreover, we discuss some aspects of the numerical solution of such equations.

Some recent results on Banach space geormetry

When, Where

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Primary AMS Classification: 46B20

Keywords and phrases: von Neumann-Jordan constant, James constant, uniform non-squareness, uniform non-ℓ_1^n-ness, direct sums of Banach spaces

Recently many geometric constants for a Banach space X have been investigated. In particular the von Neumann-Jordan constant $C_{NJ}(X)$ and the James constant $J(X)$ are most widely treated. First we shall present some recent results on these constants $C_{NJ}(X)$ and $J(X)$, where uniform non-squareness plays an important role. Secondly we shall discuss uniform non-squareness and more generally uniform non-ℓ_1^n-ness for direct sums of Banach spaces.

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Fubini Type Theorems for L^p-moduli of Continuity

When, Where

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Primary AMS Classification: 46E30

Keywords and phrases: Moduli of continuity; sections of functions; Fubini theorem

Let $f(x,y)$ be a function in $L^p(\mathbb{R}^2)$ $(1 \le p < \infty)$ and let $\omega_1(f;\delta)_p$ be the partial modulus of continuity of f in L^p with respect to the variable x. For almost all $y \in \mathbb{R}$ the sections $f_y(x) = f(x,y)$ belong to $L^p(\mathbb{R})$. We study estimates of the L^p-modulus of continuity of f_y in terms of $\omega_1(f;\delta)_p$.

It is clear that if $\omega_1(f;\delta)_p = O(\delta)$, then $\omega(f_y;\delta)_p = O_y(\delta)$ for almost all y. However, except for this limiting case, the sections f_y may not preserve the order of the modulus of continuity $\omega_1(f;\delta)_p$. Roughly speaking, Fubini-type theorem does not hold, and the sections "lose smoothness". The problem is to give a sharp estimate of this loss.

Denote by Ω the class of all non-decreasing, continuous and bounded functions $\omega(\delta)$ on $[0,+\infty)$ satisfying the conditions

$$\omega(\delta+\eta) \le \omega(\delta) + \omega(\eta), \quad \omega(0) = 0.$$

Our main result is a sharp condition on a function $\eta \in \Omega$ in terms of $\omega_1(f;\delta)_p$ $(1 \le p < \infty)$ in order that

$$\varphi_\eta(y) = \sup_{\delta>0} \frac{\omega(f_y;\delta)_p}{\eta(\delta)} < \infty$$

for almost all $y \in \mathbb{R}$.

Gagliardo-Nirenberg inequality in some Banach function spaces

When, Where

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Primary AMS Classification: 46E30

Keywords and phrases: Gagliardo-Nirenberg inequality, variable exponent Lebesgue space, Hardy-Littlewood operator

Gagliardo-Nirenberg interpolation inequalities

$$\|\nabla^k f\|_s \le C\|f\|_q^{1-\frac{k}{m}}\|\nabla^m f\|_p^{\frac{k}{m}}$$

where $f \in C_0^\infty(\mathbb{R}^n)$, $\frac{1}{s} = (1-\frac{k}{m})\frac{1}{q} + \frac{k}{m}\frac{1}{p}$, and $\nabla^l f = \{D^\alpha f\}_{|\alpha|=l}$, play an important role in the apriori estimates in linear and nonlinear PDEs and their applications to the regularity theory. Recently the Gagliardo-Nirenberg inequalities have been sharpened and extended in different directions. We prove analogies of the classical Gagliardo-Nirenberg inequalities when usual L^p norms are replaced by variable $L^{p(\cdot)}$ (Orlicz or Lorentz) norms.

On approximation of convex bodies by convex algebraic level surfaces

When, Where

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Primary AMS Classification: 41A17

Keywords and phrases: convex level surfaces of algebraic polynomials, convex bodies, rate of approximation

In this talk we consider the problem of approximation of convex bodies in $\mathbb{R}^d$ by level surfaces of convex algebraic polynomials. P.C. Hammer [1] verified that any convex body in $\mathbb{R}^d$ can be approximated by a level surface of a convex algebraic polynomial. In [2] a quantitative version of Hammer's approximation theorem was given by showing that the order of approximation of convex bodies by convex algebraic level surfaces of degree n is bounded from above by $c\frac{\log n}{n}$. In this talk we discuss further improvements of this approximation result by showing an upper bound of order $\frac{1}{n}$. Moreover, it turns out that this bound is sharp, in general.

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Alternate signs Banach-Saks property and real interpolation of operators

When, Where

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Primary AMS Classification: 46B70

Keywords and phrases: Alternate signs Banach-Saks property, real interpolation method, spreading model.

In the space of bounded linear operators acting between Banach spaces we define a seminorm vanishing on the subspace of operators having the alternate signs Banach-Saks property. We obtain logarithmically convex-type estimates of the seminorm for operators interpolated by the Lions-Peetre real method. In particular, the estimates show that the alternate signs Banach-Saks property is inherited from a space of an interpolation pair (A_0, A_1) to the real interpolation spaces $A_{\theta,p}$ with respect to (A_0, A_1) for all $0 < \theta < 1$ and $1 < p < \infty$.

A Nyström Method for Cauchy Singular integral equations with negative index

When, Where

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Primary AMS Classification: 65R20

Keywords and phrases: Cauchy singular integral equations, Fredholm integral equations

This talk deals with the numerical treatment of the following type of Cauchy singular integral equations with constant coefficients

$$aF(y) + \frac{b}{\pi}\int_{-1}^{1}\frac{F(x)}{x-y}dx + \mu\int_{-1}^{1}k(x,y)F(x)dx = g(y), \quad |y|<1, \tag{1}$$

where $a, b \in \mathbb{R}$ are constants such that $a^2 + b^2 = 1$, $b \neq 0$, $\mu \in \mathbb{R}$ and k and g are given functions on $(-1,1)^2$ and $(-1,1)$, respectively. The function F is the unknown of the equation and it is usually represented in the following form

$$F(x) = f(x)v^{\alpha,\beta}(x),$$

where $f(x)$ is a smooth function and $v^{\alpha,\beta}(x) = (1-x)^\alpha(1+x)^\beta$ is a Jacobi weight. The exponents $-1 < \alpha, \beta < 1$ are given by

$$\alpha = M - \frac{1}{2\pi i}\log\left(\frac{a+ib}{a-ib}\right), \quad \beta = N + \frac{1}{2\pi i}\log\left(\frac{a+ib}{a-ib}\right),$$

with M and N integers chosen so that $\chi = -(\alpha+\beta) = -(M+N) = -1$.

In this paper we consider equations of the form (1) with $\chi = -1$ in spaces of continuous functions with uniform norm.
The procedure we propose here consists in reducing (1), under suitable assumptions on k and g, to an equivalent regularized Fredholm integral equation and in solving the latter by a numerical Nyström-type method. This approach permits to solve a determined and well conditioned linear system and to construct an interpolating function convergent to the exact solution of the original problem.

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Kantorovich-type operators associated with positive projections and their limit semigroup

When, Where

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Vita Leonessa (speaker), *University of Basilicata, Italy*

Primary AMS Classification: 41A36

Keywords and phrases: Positive projection. Positive approximation process. Kantorovich operator. Degenerate second-order elliptic differential operator. Feller semigroup. Approximation of semigroup.

In this talk we discuss the main properties of a new sequence $(C_n)_{n\geq 1}$ of positive linear operators acting on the space $C(K)$ of all continuous functions defined on a convex compact subset K of $\mathbb{R}^N$ $(N \geq 1)$. These operators are defined by means of a positive projection on $C(K)$ and a sequence of probability Borel measures on K.

They represent a natural generalization of the operators introduced and studied in [1] and, when K is equal to $[0,1]$, they turn into the Kantorovich operators.

After showing some examples, we prove that the sequence $(C_n)_{n\geq 1}$ is an approximation process on $C(K)$ and we present some estimates of the rate of convergence. Some qualitative properties of C_n's are also obtained.

Furthermore we show that by means of iterates of the operators C_n it is possible to approximate a uniquely determined Feller semigroup whose generator is the closure of a degenerate second-order elliptic differential operator on K. As a consequence it is possible to obtain a representation formula for the solutions to suitable diffusion problems on K and also to infer some spatial regularity properties of such solutions.

All these results are contained in [2].

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Point of continuity property and boundedly complete sequences

When, Where

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Primary AMS Classification: 46B20

Keywords and phrases: Poinf of continuity property, boundedly complete basic sequences

Recall that a Banach space is said to have the point of continuity property (PCP) provided every non-empty closed and bounded subset admits a point of continuity of the identity map from the weak to norm topologies. It is known that Banach spaces with Radon-Nikodym property, including separable dual spaces, satisfy PCP, but the converse is false. The PCP has been characterized for separable Banach spaces, and this characterization implies that Banach spaces with PCP have many boundedly complete basic sequences, and so many subspaces which are separable dual spaces. As PCP is separably determined, that is, a Banach space satisfies PCP if every separable subspace has PCP, it is natural looking for a sequential characterization of PCP. In this sense, it has been proved by H. Rosenthal that every semi-normalized basic sequence in a Banach space with PCP has a boundedly complete subsequence. The converse of the above result is false in general, but it is open for Banach spaces not containing ℓ_1. The goal of this note is to prove that there exist Banach spaces failing PCP and not containing ℓ_1 such that every semi-normalized basic sequence has a boundedly complete subsequence. Concretely, the space B_∞, the natural predual of the space JT_∞, is the desired example.

On an impulsive fractionnal problem

When, Where

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N. E. Tatar, *King Fahd University, Saoudi Arabia*

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Primary AMS Classification: 35K55

Keywords and phrases: Impulsive integral equation of fractionnal derivatives, existence and unicity of solution

In this talk, we resolve an impulsive fractional problem. We state the necessary and sufficient conditions for the existence and the unicity of the solution. We use essentially some well known results in the case of the differential equations with normally derivatives. the results are obtained using the notion of fractional derivatives in the Riemann-Liouville sence.

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Structure of Cesàro function spaces

When, Where

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Cesàro function spaces $Ces_p(I)$ on both $I = [0,1]$ and $I = [0,\infty)$ are the classes of Lebesgue measurable real functions f on I such that

$$\|f\|_{C(p)} = \left[\int_I \left(\frac{1}{x}\int_0^x |f(t)|\,dt\right)^p dx\right]^{1/p} < \infty \quad \text{for} \quad 1 \le p < \infty,$$

and
$$\|f\|_{C(\infty)} = \sup_{x\in I,\ x>0} \frac{1}{x}\int_0^x |f(t)|\,dt < \infty \quad \text{for} \quad p = \infty.$$

The Cesàro function spaces $Ces_p(I)$ for $1 < p < \infty$ are separable, strictly convex and not symmetric. They, in the contrast to the sequence spaces, are not reflexive and do not have the fixed point property (cf. [1]).

Cesàro sequence spaces ces_p for $1 < p < \infty$ are defined as the set of all real sequences $x = \{x_k\}$ such that $\|x\|_{c(p)} = \left[\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{k=1}^{n}|x_k|\right)^p\right]^{1/p} < \infty$. They are separable, reflexive, not symmetric and not B-convex Banach spaces (cf. [3], [4]) but they have the fixed point property.

The structure of the Cesàro function spaces $Ces_p(I)$ is investigated. Their dual spaces, which equivalent norms have different description on $[0,1]$ and $[0,\infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \le q \le \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0,1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0,1]$ and $[0,\infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0,1]$ an uncomplemented space which is isomorphic to l^2, but they span another space in $C(\infty)$.

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Problems on the curves in a complex plane related with classic approximation theorems

When, Where

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Primary AMS Classification: 30E10

Keywords and phrases: Approximation by Polynomial, Inequalities in approximation

We consider the following model case of Jackson, Jackson-Bernstein, Bernstein and Nikolsky-Timan-Dzjadyk classic theorems.

Theorem (Jackson's). *Let* $f \in Lip_{[a,\, b]}\alpha,\ (0 < \alpha \le 1)$*, then*

$$E_n\left(f,\ [a,\ b]\,\right) \le \frac{const}{n}.$$

Theorem (Jackson-Bernstein). *In order that*

$$f \in Lip_{[0,\, 2\pi]}\alpha,\ (0 < \alpha < 1) \Leftrightarrow E_n\left(f;\ [0,\ 2\pi]\,\right) \le \frac{const}{n^{\alpha}}.$$

Theorem (Bernstein). *Let* $f_0\left(\theta\right) = f\left(\cos\theta\right) = f(x),\ x = [-1, 1].$

In order that

$$f_0\left(\theta\right) \in Lip_{[0,\, 2\pi]}\alpha,\ (0 < \alpha < 1) \Leftrightarrow E_n\left(f;\ [-1,\ 1]\,\right) \le \frac{const}{n^{\alpha}}.$$

Theorem (Nikolsky-Timan-Dzjadyk). *In order that*

$$f \in Lip_{[-1,1]}\alpha \quad (0 < \alpha < 1) \Leftrightarrow \exists P_n$$

for which $\forall x \in [-1, 1]$

$$|f(x) - P_n(x)| \le const \left(\frac{\sqrt{1-x^2}}{n} + \frac{|x|}{n^2} \right)^{\alpha}.$$

The problems we are interested may be formulated in a general form:
What necessary and sufficient conditions should (must) satisfy a class of curves in a complex plane, in order appropriate classic theorem of approximation be fulfilled (true) on it.

These problems are urgent both in the metric C and in the metric L_p . Notice that the problem related with Jackson-Bernstein theorem was formulated by J.Walsh and the problem related with Jackson theorem by D.Newman.

Orthogonal Polynomials on the Unit Circle. Spectral Transformations and Integrable Systems

When, Where

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Primary AMS Classification: 42C05

Keywords and phrases: Integrable systems, Lax pairs, Orthogonal polynomials, Probability measures on the unit circle, Verblunsky parameters

In this talk we will present a survey about properties of orthogonal polynomials on the unit circle (OPUC) related to one-parameter deformations of the corresponding nontrivial probability measure of orthogonality μ.

First, we study the time dynamics of the Verblunsky parameters, i.e. the evaluation at $z = 0$ of such orthogonal polynomials, see [4], focussing our attention in three situations

1.- The Schur flow, which is characterized by a complex semidiscrete modified KdV equation and where a discrete analogue of the Miura transformation appears. See [2] and [3].

2.- The Uvarov flow, associated with a perturbation of the probability measure by the addition of a Dirac mass located on the unit circle.

3.- The canonical Christoffel flow, when a positive first degree trigonometric polynomial perturbation where the zero depends of the time parameter.

Second, the Lax pair for the CMV and GGT matrices (see [4]) associated with such deformations is discussed.

Finally, some open problems in the framework of spectral transformations of probability measures supported on the unit circle will be analyzed. The study of such perturbations of measures started in [1] from the point of view of the relation between the corresponding sequences of orthogonal polynomials and, as a consequence, between their GGT matrices.

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Slicely Countably Determined Banach spaces

When, Where

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Vladimir Kadets, *Kharkov National University, Ukraine*

Miguel Martn (speaker), *Universidad de Granada, Spain*

Javier Merí, *Universidad de Granada, Spain*

Varvara Shepelska, *Kharkov National University, Ukraine*

Primary AMS Classification: 46B20

Keywords and phrases: Radon-Nikodm property, Asplund spaces, containing of ℓ_1, numerical radius, numerical index, Daugavet equation

A (separable) Banach space X is slicely countably determined if for every closed convex bounded subset A of X there is a sequence of slices (S_n) such that each slice of A contains one of the S_n. SCD-spaces form a joint generalization of spaces not containing ℓ_1 and those having the Radon-Nikodm property. We present many examples and several properties of this class. We give some applications to Banach spaces with the Daugavet and the alternative Daugavet properties, lush spaces and Banach spaces with numerical index 1. The talk is based in the recent paper [1].

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Ergodicity for operators generating a bounded resolvent family on Banach spaces

When, Where

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Primary AMS Classification: 47D03

Keywords and phrases: dual space, ergodicity, C_0-semigroup

Let X be a Banach space and A a closed linear operator on X, $\overline{D(A)} \subset X$. We assume that $(0,\infty) \subset \rho(A)$ and that the operators $\alpha(\alpha I - A)^{-1}, \alpha > 0$ are uniformly bounded. We consider sufficient conditions for the ergodicity of A, i.e. conditions which imply the decomposition $X = \mathrm{Ker}(A) \oplus \overline{AD(A)}$. The Banach space is assumed to be complemented in its second dual with a projection P. We prove that an assumption in terms of P and a constant K such that $\|A(\alpha I - A)^{-1}\| \leq K$ for all $0 < \alpha \leq \alpha_0$, for some $\alpha_0 > 0$, implies ergodicity. Some examples are given. In case of a stronger assumption $\|(A(\alpha I - A)^{-1})^n\| \leq K$ for all $\alpha > 0, n \in \mathbb{N}$ it is known that if A is bounded (E. Scheffold 2004) then A^{-1} generates on $\overline{AX}$ a bounded C_0-semigroup.

C_0-semigroups and modified Szász-Mirakjan operators

When, Where

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Sabina Milella (speaker), *University of Bari, Italy*

Primary AMS Classification: 41A36

Keywords and phrases: Positive semigroup, parabolic equation, weighted continuous function space, approximation by positive operators.

Consider the degenerate differential operator

$$Lu(x) = xu''(x) + \beta(x)u'(x) + \gamma(x)u(x) \quad (x > 0),$$

where $\beta \in C(]0,+\infty[)$ and $\gamma \in C_b([0,+\infty[)$, defined on a subspace $D(L)$ of $C_*([0,+\infty[)$ or of the weighted space E_m^0, which includes a kind of Wentzell-type conditions at the boundary.

Here $E_m^0 := \left\{u \in C([0,+\infty[) \mid \lim_{x\to+\infty} \frac{u(x)}{1+x^m} = 0\right\}$, for $m \geq 0$.

We prove that

- $(L, D(L))$ generates a positive C_0-semigroup $(T(t))_{t\geq 0}$ on $C_*([0,+\infty[)$ and on E_m^0;

- $(T(t))_{t\geq 0}$ can be approximated (with respect to the uniform norm and the weighted norm) by iterates of suitable positive linear operators.

In order to obtain such an approximation formula, we introduce and study the following modification of the classical Szász-Mirakjan operators.

$$S_n^*(f)(x) := \sum_{k=0}^{+\infty} e^{-nx} \frac{(nx)^k}{k!} \left(1 + \frac{\gamma(k/n)}{2n}\right) f\left(\frac{k}{n} + \frac{\beta(k/n)}{2n}\right),$$

$(f \in E_m^0, x \geq 0)$.

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Integral equation formulations of some 2D contact problems

When, Where

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Primary AMS Classification: 45A05, 45E05, 74K20, 74M15

Keywords and phrases: Integral Equations, Contact problems

In this talk we analyze some difficulties encountered when a contact problem between a Kirchhoff (thin) plate, discontinuously supported along arcs or segments, is formulated in terms of integral equations.

Two main approaches have been employed for solving these contact problems: the differential and the integral ones.

According to the PDE formulation, the plate deflection is assumed as the primary unknown, which satisfies the biharmonic equation. The boundary conditions along the supported edge are imposed in terms of the plate deflection and its derivatives, and the reaction forces and couples (or moments) are subsequently computed from the deflection field.

In the integral equation approach, a preventative selection of the suitable types of equivalent reaction forces and couples is effected, and the plate deflections are formulated as integral transformations (based upon suitable Green functions) of the unknown reaction forces. Each reaction component, through the associated integral transform having a specific kernel, defines a corresponding plate deflection component. In particular the shear force and the twisting moment have different integral transform kernels: the second one is the derivative of the first one. Therefore, to consider a reaction composed only of a shear force, or of a twisting moment, or of the sum of the two, is not in general the same thing.

If the contact reactions are correctly chosen, then this integral representation satisfies the biharmonic equation as well as all the contact problem boundary conditions, except those expressing the requirement that the plate deformed shape matches with the support profile. These compatibility conditions are then reduced to integral equations whose solution defines the unknown reaction forces. Once these have been determined, the plate deflection can be computed everywhere by means of its integral representation.

The force reactions, being the solution of (at most singular) integral equations, must generally respect functional restrictions, which exclude non integrable functions. Therefore it follows that a preventative knowledge of the singularity strength of the contact reaction is necessary to choose whether to include a shear force and/or a twisting moment among the contact reactions. Indeed, if the components of the contact reaction are not chosen properly (i.e. they are assumed to be integrable even though they are not), the plate deflection given by its integral transform representation turns out to be not a solution of the original problem, in the sense that the boundary profile defined by this representation does not match the support profile.

To illustrate these concepts, we describe their application to three different contact problems: an infinite plate resting on a finite segment support, and two distinct cases of circular plates supported along two antipodal edge arcs. In particular we discuss existence, uniqueness and endpoint behaviors of the chosen reaction forces and the role of these in defining integral equations having unique integrable solutions.

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Generalized Lebesgue constants for Fourier-Jacobi partial sums

When, Where

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Primary AMS Classification: 41A25

Keywords and phrases: Fourier-Jacobi partial sums, Lebesgue constants

Let $L_{p,A,B}$, $A, B > -1$, be the space of measurable on $[-1, 1]$ functions f, such that $||f||_{p,A,B} = ||f(1-\cdot)^A(1+\cdot)^B||_p < \infty$, and let $S_n^{\alpha,\beta}(f;x)$ be the partial sum of the function f Fourier-Jacobi series. Denote by

$$D^{\alpha,\beta}_{n,p,\gamma,\delta,A,B} = \sup_{\|f/\rho(n,\gamma,\delta)\|_{p,A,B}\leq 1} \|S_n^{\alpha,\beta}(f)\|_{p,A,B},$$

where $\rho(n,\gamma,\delta,x) = (\sqrt{1-x}+1/n)^\gamma(\sqrt{1+x}+1/n)^\delta$, $\gamma,\ \delta \in \mathbf{R}$. Then $D^{\alpha,\beta}_{n,p,\gamma,\delta,A,B}$ is called generalized Lebesgue constants. If $\gamma = \delta = 0$, these constants are the classical Lebesgue constants. V.P.Motornyi proved the boundedness of $D^{0,0}_{n,p,\gamma,\gamma,0,0}$, if $\gamma > 2/p - 3/2,\ p \in (1;4/3]$. This allowed to prove that Fourier-Legendre partial sums provide the best order approximation in L_p $(1 < p \le 4/3]$ for functions with sufficiently good differential-difference properties. Remark, the classical Lebesgue constants have the power growth in these cases. For the general case we prove, say, **Theorem 1.** *Let* $\alpha,\beta > -1/2,\ \ \mu = (\alpha+1)(1/2-1/p) \le -1/4; \nu = (\beta+1)(1/2-1/p) \le -1/4, A = \alpha/p, B = \beta/p\ , 1/p + 1/q = 1$. *Then*

$$D^{\alpha,\beta}_{n,p,\gamma,\delta,\alpha/p,\beta/p} \asymp \begin{cases} C_{\gamma,\delta}, & if \gamma > -2\mu - 1/2,\ \delta > -2\nu - 1/2, \\ C_{\mu,\nu} \ln^{1/q}(n+1), & if \gamma = -2\mu - 1/2 > 0\ \ or\ \ \delta = -2\nu - 1/2 > 0, \\ C \ln(n+1), & if \gamma = \delta = 0;\ \mu = \nu = -1/4. \end{cases}$$

Theorem 1 and 2 supply the investigations of H. Pollard, J. Newman, W. Rudin, B. Mackenhoupt, V.M. Badkov, N.M. Kazakova and orthers on the mean convergence of Fourier-Jacobi series.

Degenerate Second Order Differential Operators on Weighted Continuous Function Spaces

When, Where

Francesco Altomare, *University of Bari, Italy*

Graziana Musceo (speaker), *University of Bari, Italy*

Primary AMS Classification: 47D07

Keywords and phrases: Degenerate second order differential operator, Positive semigroup

In this talk we present some recent generation results of strongly continuous positive semigroups on weighted continuous function spaces obtained in [1] and [2]. More precisely we will consider weighted spaces of the form

$$E^w(J) := \{f \in C(J) :\ wf \in E(J)\} \tag{1}$$

where J is an open real interval, w is a weight function on J, i.e. $w \in C(J)$ and $w(x) > 0$ for every $x \in J$, and

$$E(J) := \left\{ f \in C(J) : \text{there exists } \lim_{x \to r_i} f(x) \in R \text{ for every } i = 1, 2 \right\} \tag{2}$$

where $r_1 := \inf J \in \mathbb{R} \cup \{-\infty\}$ and $r_2 := \sup J \in \mathbb{R} \cup \{+\infty\}$.
In this framework we will consider degenerate second order differential operators of the form

$$Lu := \alpha u'' + \beta u' + \gamma u \tag{3}$$

with continuous coefficients on J.
Our purpose is to find boundary conditions which define a suitable domain $D(L)$ such that $(L, D(L))$ is the generator of a strongly continuous positive semigroup on $E^w(J)$.

References

[1] Francesco Altomare and Graziana Musceo, *Positive semigroups generated by degenerate second-order differential operators,* Funkcialaj Ekvacioj, 51 (2008), 431-458.

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On the coefficients Fourier and absolute summability almost everywhere of series with respect to block orthonormal systems

When, Where

Givi Nadibaidze, *Tbilisi State University, Georgia*

Primary AMS Classification: 42C20

Keywords and phrases: Block-Orthogonal system, Weyl multipliers

Block-orthonormal systems were introduced by Gaposhkin . He proved, that the Menshov-Rademacher‘s theorem and the strong law of large numbers are valid for such systems in certain conditions. There were obtained some results on convergence and summability of series with respect to block-orthonormal systems.In particular, Menshov-Rademacher's and Gaposhkin's theorems were generalized and the exact Weyl multipliers for the convergence and summability almost everywhere of series with respect to block-orthogonal systems were established.

Nadibaidze obtained the sufficient conditions on the length of blocks guaranteeing $(C, \alpha)(-1 < \alpha < 0)$-summability almost everywhere of block-orthogonal series.

Naw we established Weyl multipliers for the absolute summability almost everywhere of series with respect to block-orthonormal systems.

We also considered some questions connected with coefficients Fourier of functions with respect to block orthonormal systems. In particular, let the sequence $\{N_k\}$ be fixed and $\Delta_k = (N_k, N_{k+1}], k = 1, 2, \ldots$.

We introduce $k(n) = \max\{k : N_k < n\}$.

Take $\{\varphi_n\}$ Δ_k-orthonormal system from $L^2(0,1)$, then for coefficients Fourier of function from $L^2(0,1)$ we have

$$\overline{\lim_{n\to\infty}} \frac{1}{k(n)} \sum_{i=1}^{n} c_i^2 \leq \|f\|_2^2$$

.

Approximation by Nörlund means of Walsh-Kaczmarz-Fourier series

When, Where

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Primary AMS Classification: 42C10

Keywords and phrases: Walsh group, Walsh system, Walsh-Kaczmarz system, Walsh-Fourier series, Walsh-Kaczmarz-Fourier series, Nörlund means, approximation.

In this talk we would like to investigate the rate of the approximation by Nörlund means of Walsh-Kaczmarz-Fourier series of a function in L^p $(1 \leq p \leq \infty)$. We will investigate the rate of the approximation by this means, in particular , in $\mathrm{Lip}(\alpha, p)$, where $\alpha > 0$ and $1 \leq p \leq \infty$. In case $p = \infty$ by L^p we mean C_W, the collection of the uniformly W-continuous functions. In special cases, we obtain the earlier result by Skvortsov [2]. Earlier results on the Walsh-Fourier series was given by Móricz and Siddiqi [1].

After this we define the Nörlund means of cubical partial sums of Walsh-(Kaczmarz)-Fourier series of a function in L^p.

Our main theorems state that the approximation behavior of the two-dimensional Walsh-(Kaczmarz)-Nörlund means defined by us is soo good as the approximation behavior of the one-dimensional Walsh-(Kaczmarz)-Nörlund means.

References

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[2] V.A. Skvortsov, *On Fourier series with respect to the Walsh-Kaczmarz system,* Analysis Math. 7 (1981), 141-150.

New kernels of Sard type and applications

When, Where

Francesco Costabile, *University of Calabria, Italy*

Anna Napoli (speaker), *University of Calabria, Italy*

Primary AMS Classification: 41A80, 41A35, 41A63

Keywords and phrases: Sard, Kernel

Sard's generalization of Peano's theorem extends to the multivariate case the well known result concerning the representation of a linear functional, belonging to a wide class, in terms of a differential operator. By using Sard's kernel theorem it is possible to express and calculate sharp bounds for approximation errors.

In this paper new useful kernels of Sard type are derived by applying a linear functional to the two-point Taylor polynomials for real functions possessing a sufficient number of derivatives. These kernels are used to calculate error representations for some cubature and approximation formulae. In all the considered examples kernels are of one sign, thus accurate error estimates are obtained.

Bartle-Dunford-Schwartz integrability versus Bochner, Pettis, Dunford integrability

When, Where

A. Fernández, *Seville University, Spain*

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Francisco Naranjo (speaker), *Seville University, Spain*

Primary AMS Classification: 46G10

Keywords and phrases: Vector measure, integrable function, distribution function

The Lebesgue's theory of integration (1902) has been generalized in several directions. Bochner, Pettis and Dunford (1933) developed a theory of integration in order to integrate vector valued functions with respect to a positive measure. On the other hand, Bartle, Dunford and Schwartz (1955) developed a theory of integration in order to integrate scalar functions with respect to vector measures. In this talk we study some relationships between these two types of generalization.

Polynomial approximation with Pollaczek-type weights

When, Where

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Incoronata Notarangelo (speaker), *Università degli Studi della Basilicata, Italy*

Primary AMS Classification: 41A10

Keywords and phrases: Approximation by Polynomials, Jackson theorem

The polynomial approximation of functions having algebraic singularities at the endpoints of the interval $[-1,1]$ has been extensively studied in [1]. In order to approximate functions defined on $(-1,1)$ and increasing exponentially at ± 1, we consider we consider a Pollaczek-type weight, for instance of the form

$$w(x) = \mathrm{e}^{-(1-x^2)^{-\alpha}}, \qquad \alpha > 0\,.$$

Revisiting some results due to D.S. Lubinsky [2], we introduce a new modulus of smoothness and the related K–functional. Then we prove a Jackson theorem in weighted L^p-spaces, $1 < p < \infty$, also in its weaker form.

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Isometries and Nonexpansive Mappings in 2-modular sense

When, Where

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Primary AMS Classification: 47S20

Keywords and phrases: Isometry, Nonexpansive Mapping, Modular

For a real vector space X of dimension greater than one, a real valued function $\rho(\cdot,\cdot)$ on X^2 satisfying the following properties is a 2-modular on X, for all $x, y, z \in X$:

1. $\rho(x, y) = 0$ if and only if x, y are linearly dependent,
2. $\rho(x, y) = \rho(y, x)$,
3. $\rho(-x, y) = \rho(x, y)$,
4. $\rho(x, \alpha y + \beta z) \leq \rho(x, y) + \rho(x, z)$, for any nonnegative real numbers α, β with $\alpha + \beta = 1$.

And the pair (X, ρ) is a 2-*modular space*. In this talk, we aim to discuss on isometries and nonexpansive mappings defined on 2-modular spaces.

Observations on Ideals in Banach spaces

When, Where

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Primary AMS Classification: 46B20

Keywords and phrases: Ideals, Hahn-Banach extension operator

Recall that a (not necessarily) closed subspace X of the Banach space Y is called an *ideal* if there is a norm-one projection $P : Y^* \to Y^*$ with kernel $X^\perp$. When X is an ideal we have $Y^* = X^\perp \oplus P(Y^*)$, where $P(Y^*)$ is isometrically isomorphic to X^*. The concept of an ideal goes back to a seminal paper of Godefroy, Kalton and Saphar published in Studia Mathematica in 1993.

When X is an ideal with projection P, there are always an associated Hahn Banach extension operator $\Phi_P : X^* \to Y^*$ and an extension of the natural embedding k_x of X into X^{**} to an operator $T_P : Y \to X^{**}$.

In the first part of my talk I will give a quick overview of how various properties of P, Φ_P and T_P are reflected in each other. Next I will present some observations on strict ideals (an ideal is strict if $P(Y^*)$ is a norming set). Finally, I will turn to a new type of ideals, Tauberian ideals (defined by the property that $\ker P$ be reflexive) and to present what we know about Tauberian ideals so far.

Some Variations of the Bohman-Korovkin Theorem

When, Where

Özlem G. Atlıhan, *Ankara University, Turkey*

H. Gül İnce, *Gazi University, Turkey*

Cihan Orhan (speaker), *Ankara University, Turkey*

Primary AMS Classification: 41A25

Keywords and phrases: Matrix summability method, summation process, statistical convergence, Korovkin type theorem

Considering bounding functions Lomelí and García, in [2], proved some variations of the Bohman-Korovkin theorem on convergence of positive linear operators on a space of continuous functions.

In this talk we will discuss some extensions of this result by using either statistical convergence [1] or summation process which includes both convergence and almost convergence[3]. Since statistical convergence and almsot convergence methods are incompatible [4], we conclude that these methods can be used alternatively to get some approximation results.

References

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[2] H. E. Lomelí and C. L. García, *Variations on a theorem of Korovkin,* Amer. Math. Montly, 113 (2006), 744-750.

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[4] H. I. Miller and C. Orhan, *On almost convergent ans statistically convergent subsequences,* Acta Math. Hungarica, 93(2001), 135-151.

Numerical treatment of integrals with highly oscillatory density function

When, Where

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Primary AMS Classification: 65D32

Keywords and phrases: oscillatory, Fredholm integral equations

In this talk we approximate integrals of the following type

$$\int_a^b f(x)g(\omega x)dx$$

where $-\infty < a < b < \infty$, f is a continuous function and g is a highly oscillatory function. We prove the stability and the convergence of the used method.

Moreover we give an application to some Fredholm integral equations and we show some numerical tests.

The talk is based on joint work in progress with Maria Carmela De Bonis.

On some generalized properties in Q-topological algebras

When, Where

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Primary AMS Classification: 46H05

Keywords and phrases: Q-Algebra, Invertible Element, Topologicaly Invertible

In a unital Banach algebra A the set $G(A)$ of its invertible elements is an open set and the application $x \to x^{-1}$ from $G(A)$ onto $G(A)$ is continuous. More generally, if A is a metrizable and complete topological algebra, then the mapping $x \to x^{-1}$ is continuous if and only if $G(A)$ is a G_δ set.

We say that a topological algebra A is a Q-algebra if the set $G(A)$ is an open set. If A is a commutative Q-algebra and the mapping $x \to x^{-1}$ is continuous, i.e. it is a Waelbroeck algebra, then all the maximal ideals are closed and of codimension 1

Q-topological algebras have some interesting properties as the following: the spectrum $\sigma(x)$ is compact for every $x \in A$ and if A is commutative, then the set $\mathfrak{M}(A)$ of all non-zero linear, multiplicative and continuous functionals of A is non empty if and only if A is not a field and A is a *Gelfand-Mazur algebra*.

We will examine some properties related to Q-algebras and their relations with some other algebras. We will also provide some interesting examples.

Quantitative Results in Weighted Approximation

When, Where

Radu Păltănea, *Transilvania University Braşov, Romania*

Primary AMS Classification: 41A36

Keywords and phrases: Weighted Approximation, Rate of Convergence, Moduli of Continuity

Let ρ a weight function on the interval $[0,\infty)$ and let the weighted space $C_\rho[0,\infty) = \{f \in C[0,\infty),\ \|f\|_\rho < \infty\}$ where $\|f\|_\rho = \sup_{x\in[0,\infty)} |f(x)|$. We give estimates of degree of approximation in the space $C_\rho[0,\infty) = \{f \in C[0,\infty),\ \|f\|_\rho < \infty\}$ for positive linear operators in terms of second order weighted modulus adapted to this space.

References

[1] N. T. Amanov, *On the uniform weighted approximation by Szász-Mirakjan operators,* Anal.Math. 18 (1992), no. 3, 167-184.

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Pre Stieltjes functions

When, Where

Henrik Laurberg Pedersen, *University of Copenhagen, Denmark*

Primary AMS Classification: 44A10

Keywords and phrases: completely monotonic function, Stieltjes transform

We characterize the class $\mathcal{C}_k$ of functions f on $(0,\infty)$ for which $f(x),\ldots,(x^k f(x))^{(k)}$ are completely monotonic for given k. In the limit we obtain the well-known characterization of the class of Stieltjes functions as those functions f defined on the positive half line for which $(x^k f(x))^{(k)}$ is completely monotonic on $(0,\infty)$ for all $k \geq 0$.

Dirac masses modifications for measures in several variables

When, Where

Teresa E. Pérez (speaker), *Universidad de Granada, Spain*

A. M. Delgado, *Universidad de Granada, Spain*

L. Fernández, *Universidad de Granada, Spain*

M. A. Piñar, *Universidad de Granada, Spain*

Yuan Xu, *University of Oregon, USA*

Primary AMS Classification: 33C50

Keywords and phrases: Dirac masses, multivariate orthogonal polynomials

In one variable, the study of orthogonal polynomials relative to measures with Dirac masses appears in a work of A. M. Krall (1981) when he studied the orthogonal polynomials that are eigenfunctions of a fourth order differential operator. The purpose of the present work is to study this problem in several variables.

Let $N \geq 1$ be a positive integer number, Λ be a symmetric and positive definite matrix of order N, and $\xi_1, \xi_2, \ldots, \xi_N$, be N fixed points on $\mathbb{R}^d$. We define the inner product

$$\langle p, q \rangle_D = \langle p, q \rangle + (p(\xi_1), p(\xi_2), \ldots, p(\xi_N))\Lambda(q(\xi_1), q(\xi_2), \ldots, q(\xi_N))^t,$$

where

$$\langle p, q \rangle = \int_G p(x)q(x)d\mu(x),$$

is a standard inner product defined on $G \subset \mathbb{R}^d$ a simply connected domain (having a nonempty interior) and $d\mu(x)$ is a measure defined on G.

Our main result relates orthogonal polynomials and reproducing kernels with respect to both inner products. In the particular case where $d\mu(x)$ denotes the classical measure on the simplex, explicit representations and asymptotic properties are obtained.

Some new developments concerning characterizations of weighted multidimensional Hardy type integral inequalities

When, Where

Lars-Erik Persson, *Lulea University of Technology, Sweden*

Abstract: First I will present some historical facts and further developments of Hardy type inequalities of interest in the understanding also of the multidimensional case. After that I will present and discuss the famous Sawyer characterization of the twodimensional case in this light. In particular, I will present some recent ideas and results by myself and my students, which can be found in the book [1]. Moreover, I will present some open questions and partial results, which can be of interest for further research.

References

[1] V.M. Kokilashvili, A. Meskhi and L.E. Persson, *Weighted Norm Inequalities for Integral Transforms with Product Kernels,* Nova Science Publishers, Inc, 2009

Two generalizations of the Bernstein polynomials that involve the q-integers

When, Where

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Primary AMS Classification: 41A10

Keywords and phrases: Bernstein polynomials, q-integers, rational functions

This paper is concerned with the generalization of the Bernstein polynomials introduced by Alexandru Lupaş in 1987 and the generalization introduced in 1996 by the author of this paper. The first of these generalizations is a rational function and the second one is a polynomial. Both generalizations involve the q-integers. Many authors have written about the latter generalization, and it is hoped that this paper will encourage further interest and work on Lupaş's generalization, which has been surprisingly neglected.

Full characterization of the best Lipschitz constants of all iterations of mean lipschitzian mappings

When, Where

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Primary AMS Classification: 47H10

Keywords and phrases: Lipschitzian mappings, contractions, nonexpansive mappings.

Let (M, ϱ) be a metric space and let $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ be a *multi-index* satisfying $\alpha_1 > 0, \alpha_n > 0, \alpha_i \geq 0$, $i = 2, ..., n-1$ and $\sum_{i=1}^{n} \alpha_i = 1$. The number n shall be referred to as the *length* of the multi-index α. A mapping $T : M \to M$ is said to be α*-Lipschitzian* for the constant $k \geq 0$ if for every $x, y \in M$ we have,

$$\sum_{i=1}^{n} \alpha_i \varrho(T^i x, T^i y) \leq k \varrho(x, y).$$

When the multi-index α is not explicitly specified we will refer to such a mapping as a *mean Lipschitzian* mapping. The aim of my talk is to present a sharp evaluation of Lipschitz constants of all iterations of α-Lipschitzian mappings when the multi-index has length $n = 2$. In this special case the above formula takes the form: $\alpha_1 \varrho(Tx, Ty) + \alpha_2 \varrho(T^2 x, T^2 y) \leq k \varrho(x, y)$. Moreover an example of linear mapping for which each iteration has the biggest possible Lipschitz constant is given.

References

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Approximation of evolution equations

When, Where

Sergey Piskarev, *Lomonosov Moscow State University, Russia*

Primary AMS Classification: 37L65

Keywords and phrases: Semigroups of operators, general approximation scheme

This paper is devoted to the numerical analysis of abstract semilinear parabolic problem

$$u'(t) = Au(t) + f(u(t)), u(0) = u^0, \tag{1}$$

in some general Banach space E. We are developping a general approach to establish discrete dichotomy in a very general settings. It is well-known fact (see [1-3]) that the phase space in the neighborhood of the hyperbolic equilibrium of (1) can be split in a such way that the original initial value problem is reduced to initial value problems with exponential decaying solutions in opposite time direction. We use the theory of regular approximation principle to show that such a decomposition of the flow persists under rather general approximation schemes. The main assumption of our results is regular convergence of infinitesimal generators in some half-plane. Such assumption is naturally satisfied for example for operators with compact resolvents and can be verified for finite element as well as finite difference methods.

References

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Efficient reconstruction of functions on the sphere from scattered data

When, Where

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Primary AMS Classification: 65TXX

Keywords and phrases: two-sphere, quadrature, nonequispaced fast spherical Fourier transform, interpolation, approximation, NFFT, FFT

Recently, fast and reliable algorithms for the evaluation of spherical harmonic expansions have been developed. The corresponding sampling problem is the computation of Fourier coefficients of a function from sampled values at scattered nodes.

We consider a least squares approximation as well as an interpolation to the given data. Our main result is that the rate of convergence of the two proposed iterative schemes depends only on the mesh norm and the separation distance of the nodes. In conjunction with the nonequispaced FFT on the sphere, the reconstruction of N^2 Fourier coefficients from M reasonably distributed samples is shown to take $\mathcal{O}(N^2 \log^2 N + M)$ floating point operations. Furthermore we discuss the computation of quadrature weights for scattered nodes on the sphere, which are exact for spherical polynomials of high degree N. We present a worst case analysis as well as results based on probabilistic arguments. Numerical results support our theoretical findings.

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Constructing separated sequences in Banach spaces

When, Where

Stanisław Prus, *M. Curie-Skłodowska University*, *Poland*

Primary AMS Classification: 46B20

Keywords and phrases: Kottman constant, Diestel's problem, spreading model

In the book "Sequences and series in Banach spaces" J. Diestel posed the following problem: for which infinite dimensional Banach spaces X one can find $c > 1$ such that every infinite dimensional subspace Y of X contains a sequence (y_n) of norm-one vectors for which

$$\inf\{\|y_i - y_j\| : i \neq j\} \geq c.$$

We give partial answers to this question. In particular we prove that if an infinite dimensional space X admits an equivalent nearly uniformly convex norm or if c_0 is not finitely representable in X, then X has the property formulated by Diestel.

Continued Fractions and Birth and Death Processes

When, Where

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Primary AMS Classification: 60J80

Keywords and phrases: Continued Fractions, Orthogonal Polynomials

An important sub-class of Markov chains with continuous time parameter space is birth and death processes (BDPs), whose state space is the non-negative integers. These processes are characterized by the property that if a transition occurs then this transition leads to a neighbouring state.

Birth and death processes are widely used in a variety of physical, biological, engineering, communication and finance fields. Connections between birth and death processes, continued fractions and orthogonal polynomials are well-known.

We consider a BDP on {0,1,2,...,N} with birth parameters, λ_{i-1} and death parameters μ_i, $i = 1, 2, ..., N$. Let α be the least eigenvalue (ie., eigenvalue with largest absolute value) associated with this BDP. We generate another BDP on {0,1,2,...,2N} using the previous birth and death parameters and α. We illustrate with an example from the continued fraction expansion of tan(kz). The largest eigenvalue in finite BDP is useful to obtain the speed of convergence towards the stationarity of the process. Interestingly here, the least eigenvalue plays a significant role.

C_0 - semigroups and iterates of positive linear operators: asymptotic behaviour

When, Where

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Primary AMS Classification: 41A36

Keywords and phrases: Positive linear operators, C_0-semigroups, iterates, asymptotic behaviour, rates of convergence

In 1989, the year of the first Maratea Conference on Functional Analysis and Approximation Theory, F. Altomare published very general results concerning the approximation of certain C_0-semigroups in terms of suitable iterates of positive linear operators. Since then, this area of research is under an active study.

Several results (mainly obtained by Altomare and his school) were published and new problems continue to appear; they involve qualitative properties of the approximated semigroups, the asymptotic behaviour, rates of convergence, the connection with the associated Markov processes and stochastic equations, the approximation of the resolvents. Some results and problems were presented at the successive Maratea FAAT Conferences.

This lecture is mainly devoted to the connection between the asymptotic behaviour of the approximated semigroups and the overiterated positive linear operators. We shall be concerned with

(i) the characterization of the positive linear operators whose iterates converge to the canonical projection on $C(K)$, K a Bauer simplex;

(ii) the overiterates of certain operators on $C(X)$, X a compact convex set;

(iii) the asymptotic behaviour of the semigroup approximated by the Bernstein-Schnabl operators;

(iv) the asymptotic behaviour of the Fleming-Viot semigroups;

(v) the connections between overiterates, diffusions, and Markov chains.

Several examples and applications will be given, involving classical semigroups and classical sequences of positive linear operators.

Regularization methods for an abstract inverse elliptic problem with Dirichlet conditions

When, Where

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Primary AMS Classification: 35R30.

Keywords and phrases: Inverse problems, elliptic problems, data completion.

In the abstract setting, let H be a complex infinite dimensional Hilbert space with the scalar product $(\cdot,\cdot)$ and the norm $\|\cdot\|$, and let $A:\mathcal{D}(A)\subset H\to H$ be a positive-definite, self-adjoint operator.

We consider the boundary value problem for the abstract elliptic equation with Dirichlet conditions

$$u''(t) - Au(t) = 0, \quad 0 < t < T, \tag{1}$$

$$u(0) = \varphi, \quad u(T) = 0 \tag{2}$$

The main goal of this study is to find the boundary condition $u(0)$ on the basis of the known boundary condition at $t = \tau \in (0,T)$, i.e.,

$$\mathcal{B}(u) = u(\tau) = h. \tag{3}$$

Our inverse problem is to determine $u(0) = \varphi$ knowing $\mathcal{B}(u) = h = \mathcal{K}\varphi$.

More precisely we want to study properties of the map $\mathcal{K}$:

1. Injectivity of $\mathcal{K}$ (identifiability);
2. Continuity of $\mathcal{K}$ and its inverse if it is exists (stability);
3. What is the range of $\mathcal{K}$ (admissible sets $\subseteq R(\mathcal{K})$);
4. Formula to recover φ from h (reconstruction).

Quasi-interpolant operators based on trivariate C^2 quartic box splines

When, Where

Sara Remogna, *University of Torino*, *Italy*

Primary AMS Classification: 65D07

Keywords and phrases: Trivariate Box Spline, Quasi-Interpolant Operator

In this talk we consider the space $S(X)$ generated by the integer translates of the trivariate C^2 quartic box spline defined by a set X of seven directions that forms a regular partition of the space into tetrahedra.

In $S(X)$ we study two kinds of local spline quasi-interpolants (abbr. QI) of the form

$$Qf = \sum_{\alpha \in \mathbb{Z}^3} \lambda_\alpha f B_\alpha,$$

where $\{B_\alpha, \alpha \in \mathbb{Z}^3\}$ is the family of the above spanning functions and $\{\lambda_\alpha, \alpha \in \mathbb{Z}^3\}$ is a family of local linear functionals. The first kind consists of QIs of differential type, whose coefficient functionals are linear combinations of values of f and its derivatives at the center of the support of B_α. The second one consists of QIs of discrete type, where the coefficient functionals are linear combinations of values of f at specific points in the support of B_α and are obtained by convenient discretisations of the differential QI coefficient.

Then we present some numerical examples illustrating the approximation properties of the proposed quasi-interpolants.

Split Bezoutians and inverses of symmetric Toeplitz or centrosymmetric Toeplitz-plus-Hankel matrices

When, Where

Karla Rost, *Chemnitz University of Technology*, *Germany*

Primary AMS Classification: 65F05

Keywords and phrases: Toeplitz matrix inverse, Toeplitz-plus-Hankel matrix inverse, Bezoutian

Inverses of symmetric Toeplitz or centrosymmetric Toeplitz-plus-Hankel matrices can be represented as a sum of two split Bezoutians, one of (+)-type, the other of (−)-type. Ideas how to develop such inversion formulas are presented. Moreover, matrix representations for split Bezoutians are developed.

A comparison of effective approximation methods by Bernstein-type operators on triangles

When, Where

Paul Sablonnière, *Insa & Université de Rennes, France*

Primary AMS Classification: 41A10

Keywords and phrases: Bernstein polynomials, multivariate approximation

Bernstein type polynomial quasi-interpolants (abbr. QI) on triangles and, more generally on simplices, have been studied by various people in the last thirty years. The main topics are saturation results for general families of functions, asymptotic expansions for convergence, or geometric developments on variation-diminishing properties. Except in some rare cases, there is no study of the *effectiveness* of approximation methods given by these operators for an eventual practical use, for example in CAGD. In this talk, I will compare effective methods provided by *quasi-interpolants* directly deduced, using the same general scheme, from the three following Bernstein operators defined in the unit triangle T with vertices $(1,0),(0,1)$ and $(0,0)$:
1) The *classical Bernstein operator* whose coefficients are discrete values of the approximated function f on the uniform n-lattice of points in T.
2) The so-called *Bernstein-Durrmeyer-Jacobi operator* whose coefficients are weighted mean values of f times the Bernstein polynomials. The Jacobi weight is $w_p(x,y,z) := x^{p_1}y^{p_2}z^{p_3}$, where $z := 1-x-y$ and $p := (p_1,p_2,p_3)$, with $p_i > -1$ for $i = 1,2,3$.
3) The extension of the preceding operator to the limit case $p := (-1,-1,-1)$, often

called the *genuine Bernstein-Durrmeyer operator* in the literature.
By using typical examples, I will try to convince the audience of the interesting practical possibilities afforded by these approximants.

Weighted Hardy inequalities in Morrey spaces

When, Where

Natasha Samko, *Universidade do Algarve, Portugal*

Primary AMS Classification: 46E30

Keywords and phrases: Morrey space, singular operator, Hardy operator, Hardy-Littlewood maximal operator, weighted estimate.

We study the one-dimensional Hardy operators in weighted Morrey spaces $\mathcal{L}^{p,\lambda}(\Gamma)$ on an interval or a curve. The case of curve is considered because of applications to singular integral equations. The weight function may be a product of a finite number of functions from Bary-Stechkin type class, with nodes on the curve. We apply the obtained weighted result for Hardy operators to prove the weighted boundedness of singular operators in Morrey spaces. We obtain sufficient conditions for the boundedness of the singular operator in terms of the Matuszewska-Orlicz indices of weights. In the case of power weights we prove that conditions on the weight are also necessary for the boundedness in the case of both Hardy and singular operators.

On the stability of collocation methods for Cauchy singular integral equations in weighted L^p spaces

When, Where

Markus Seidel, *Chemnitz University of Technology, Germany*

Primary AMS Classification: 45L05

Keywords and phrases: Cauchy singular integral equation, polynomial collocation method, stability, approximation numbers

Collocation methods w.r.t. Chebyshev nodes for Cauchy singular integral equations with piecewise continuous coefficients have been extensively studied in weighted $\mathbf{L}^2$ spaces (cf. [1], for instance). There, the stability of the collocation sequences as well as their so-called Fredholm property which is closely related to a nice asymptotic behavior of the respective singular values have been the key notions under consideration. Based on Marcinkiewicz inequalities and with the help of abstract Banach algebra techniques these methods can be extended to equations

$$a(x)u(x) + \frac{b(x)}{\pi} \int_{-1}^{1} \frac{u(y)}{y-x}\, dy = f(x)\,, \quad -1 < x < 1\,,$$

in the spaces $\mathbf{L}^p_\sigma$ with $1 < p < \infty$, the weight $\sigma(x) = (1-x^2)^{-\frac{1}{2}}$, and a, b piecewise continuous. In this setting, approximation numbers turn out to be the appropriate substitutes for the singular values. We state necessary and sufficient conditions for the stability of the collocation methods as well as results on the approximation numbers in terms of winding numbers of certain closed curves which easily arise from the coefficients a, b.

This talk is based on joint work with P. Junghanns and G. Mastroianni.

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On The Strict Best Approximation

When, Where

Fowzi A. Sejeeni, *Umm Al-qura University, Makkah, Saudi Arabia*

Primary AMS Classification: 49M25

Keywords and phrases: Strict approximation, Polya algorithm

We consider the Euclidean space $\mathbb{R}^m$ as the set of all real functions defined on a finite set T of m elements. The definition of strict approximation supported in the following: For the subspace $\mathbf{G}$ of $\mathbb{R}^m$ and f in $\mathbb{R}^m \backslash \mathbf{G}$ there exists a nonempty subset of T denoted by Ex(f) uniquely determined by f, such that, for each t in Ex(f) there exists a real constant k_t with $g(t)= k_t$ for each g in $\mathbb{P}_{\mathbf{G}}(f)$, where

$$\mathbb{P}_{\mathbf{G}}(f) = \{g \in \mathbf{G} : \|f - g\| \leq \|f - h\| \ \forall\ h \in \mathbf{G}\}.$$

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Pointwise estimates in coconvex approximation

When, Where

Igor A. Shevchuk, *Taras Shevchenko National University of Kyiv*, *Ukraine*

Primary AMS Classification: 41A29

Keywords and phrases: shape preserving approximation, algebraic polynomials, pointwise estimates

We will discuss the validity of classical Nikolskii type estimates for the coconvex (that is piecewise convex) approximation. Recall, these classical estimates belong to Timan, Dzyadyk, Freud and Brudnyi. For the monotone and convex approximations such estimates belong to Lorentz and Zeller, DeVore, Yu, Leviatan and the author. It turns out, that coconvex pointwise approximation has a few new, sometimes unexpected phenomenons, comparatively not only with unconstrained approximation, but even with other types of the shape preserving approximation.

Banach algebras of structured matrix sequences and topics in asymptotic spectral theory

When, Where

Bernd Silbermann, *TU Chemnitz, Germany*

This talk is devoted to some problems in asymptotic spectral theory for matrix sequences which can be studied by Banach algebra techniques (asymptotic behavior of approximation numbers, ε-pseudospectra, and of related spectral quantities). The first part of the talk deals with some general features of the theory (Fredholm sequences in some general Banach algebra $\mathcal{F}$, fractal subalgebras) and their consequences. Further, a special kind of subalgebras with excellent properties is singled out. These subalgebras reflect some structure of the involved matrix sequences. The second part presents (nontrivial) examples. A prominent role will play the Banach algebra constituted by finite section sequences of banddominated operators. This algebra is nonfractal and contains for instance the fractal subalgebra which is generated by finite section sequences of Toeplitz operators with generating functions from the Wiener algebra.

A General Theorem for Rational Approximation on the Extended Complex Plane

When, Where

Mehrdad Simkani, *University of Michigan-Flint, USA*

Primary AMS Classification: 30E10

Keywords and phrases: degree of approximation, region of meromorphy

Motivated by a classic work of Bernstein, linking the degree of best polynomial approximation to the region of holomorphy, we present a general theorem for rational approximation on the extended complex plane. Our result relates the degree of rational approximation to the region of meromorphy. And, it unifies the cases of free poles and prescribed poles. Particularly, we generalize theorems of Walsh, Walsh-Russell, Shen, Bagby, Saff, Bojadziev, Goncar, and Reczek-Simkani.

Ky Fans Best Approximation Theorem and Applications

When, Where

Antonio Carbone, *Università della Calabria, Italy*

Sankatha P. Singh (speaker), *Memorial University, Canada*

Primary AMS Classification: 47H10

In this survey talk we briefly present results related to the best approximation theorem of Ky Fan and its applications. The theorem is of great importance in nonlinear analysis, approximation theory, minimax theory, game theory, fixed point theory, mathematical economics and variational inequalities.

We first give results in finite dimensional case and Hilbert spaces before considering other aspects.

AAK Approximants to Functions with Branch Points

When, Where

Herbert Stahl, *TFH-Berlin / FB II, Germany*

Primary AMS Classification: 41A20

Keywords and phrases: meromorphic best approximation, asymptotic error estimates, potential theory, AAK-theory, functions with branch points.

AAK (Adamjan-Arov-Krein) approximants are a very successful concept for meromorphic best approximation on the unit circle with interesting applications in engineering and applied sciences, especially widespread is their use in control and identification of linear systems. Computationally, the concept is very efficient thanks to close connections with Hankel operators and their singular values.

We are concerned with the convergence behavior of this type of approximants to functions f that are holomorphic outside of the open unit disk $\mathbb{D}$ and have all their singularities in a compact set of capacity zero in $\mathbb{D}$. The most interesting cases arise when the function f has branch points among its nonpolar singularities.

Asymptotic error estimates, the over-convergence inside and outside of the unit circle $\mathbb{T}$, and the asymptotic distribution of the poles of the approximants are topics of primary interest in the talk.

Trigonometric Polynomials of Semi-Integer Degree Orthogonal with Respect to a Moment Functional

When, Where

Gradimir V. Milovanović, *Megatrend University, Belgrade, Serbia*

Aleksandar S. Cvetković, *University of Niš, Serbia*

Marija P. Stanić (speaker), *University of Kragujevac, Serbia*

Primary AMS Classification: 42C05

Keywords and phrases: Trigonometric Polynomial, Semi-Integer Degree, Orthogonality, Moment Functional, Recurrence Relation, Fourier Series, Reproducing Kernel

In this talk we are concerned with the algebraic properties of orthogonal trigonometric polynomials of semi-integer degree. Such orthogonal systems arise in theory of quadrature rules with maximal trigonometric degree of exactness (see [1] and [2]). We investigate theory of orthogonality with respect to a general linear functional mapping space of trigonometric polynomials to the real numbers. We prove that under certain conditions, imposed on the linear functional, a sequence of orthogonal trigonometric polynomials of semi-integer degree exists and satisfies the three-term recurrence relations when treated in the suitable matrix settings. Special attention is devoted to the case of a moment functional defined by a nonnegative Borel measure.

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Eighteen years of cooperation with Giuseppe Mastroianni - A survey

When, Where

József Szabados, *Alfréd Rényi Institute of Mathematics*, *Hungary*

Various subjects in approximation theory, like weighted polynomial approximation, interpolation processes, Shepard operators, Markov–Bernstein type inequalities, extensions of classical operators (Bernstein, Szász–Mirakyan, Kantorovich), will be discussed.

On the order of convergence of the Voronovskaja-type formulas and applications

When, Where

Cristian Tacelli, *University of Salento*, *Italy*

Primary AMS Classification: 41A36

Keywords and phrases: TrotterKato approximation theorem, Representation of semigroups, Convergence of iterates of linear operators

Starting with some quantitative estimates of the Voronovskaja-type formulas depending on the moduli of continuity of the second-order derivative and the differential operator, we extend as much as possible the class of functions for which we

can find a quantitative estimate of the convergence of suitable sequences of linear operators to the solution of an assigned evolution problem (see [1],[2]).

We also give some applications in the context of more general operators connected with suitable evolution problems.

References

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Performance of A New Numerical Solver for Differential equations based on Discontinuous Galerkin Finite Element Methods

When, Where

Helmi Temimi (speaker), *Gulf University for Sciences and Technology, Kuwait*

Slimane Adjerid, *Virginia Polytechnic Institute and State University*, *USA*

Mohamed Ayari, *National School of Engineering*, *Tunisia*

Primary AMS Classification: 65N30

Keywords and phrases: Discontinuous Galerkin Method, Rate of Convergence, Structural Model

In this paper, we implement the Discontinuous Galerkin (DG) method as a new Numerical Solver, proposed and well-analyzed for linear and nonlinear problems by Adjerid *et.al*, to study the response of a multi-story building seismically excited. In fact, we adopt the DG method to solve a system of second order ordinary differential equations. The unknowns are the time response of the inter-story displacement of each level of the building. The DG method is very appealing regarding its efficiency proven for several types of problems in terms of its robustness, stability, higher order accuracy (pointwise error is h^{p+1}, where h is the step size and p is the degree of approximation) and approximation by polynomials of different degrees in different elements. A comparison of the results obtained through the DG method and through traditional numerical schemes is conducted. The results reveal the efficacy of the DG method, which lends it as an attractive alternative instead of currently used numerical techniques.

Representative product systems

When, Where

Rodolfo Toledo, *College of Nyíregyháza, Hungary*

Primary AMS Classification: 42C10

Keywords and phrases: Vilenkin systems, Walsh-Paley system, representative product system, Fourier series, Dirichlet kernels, Fejér means, Cesàro means

The Walsh-Paley system is formed by the characters of the dyadic group, i.e., the complete product of the discrete cyclic group of order 2 with the product of topologies and measures. Vilenkin in 1947 generalized this structure studying the complete product of arbitrary cyclic groups. In Vilenkin groups the order of the cyclic groups appeared in the product can be unbounded. The methods applied in the study of these cases differ significantly from the bounded cases and in many instances we obtain different results for the same question. A natural generalization of the Vilenkin groups is the complete product of arbitrary groups, non necessarily commutative groups. In this case we use representation theory in order to obtain orthonormal systems, taking the finite product of the normalized coordinate functions of the continuous irreducible representations appeared in the dual object of the finite groups. These systems are named representative product systems.

In this talk I deal with the convergence in L^p-norm of Fourier series, Fejér means and Cesàro means of order α with respect to representative product systems, where $1 \leq p < \infty$. I also deal with the properties of the maximal values of Dirichlet kernels emphasizing the differences between commutative and noncommutative structures.

Mixed recurrence relations and interlacing theorems for orthogonal and q-orthogonal polynomials

When, Where

Ferenc Toókos (speaker), *Helmholtz Zentrum München, Germany*

Kerstin Jordaan, *University of Pretoria, South Africa*

Primary AMS Classification: 33C20

Keywords and phrases: Orthogonal polynomials, q-orthogonal polynomials, Interlacing of zeros, Separation of zeros

We study the interlacing properties of the zeros of orthogonal polynomials p_n and r_m, $m = n$ or $n-1$ where $\{p_n\}_{n=1}^{\infty}$ and $\{r_m\}_{m=1}^{\infty}$ are different sequences of orthogonal polynomials. The results obtained extend a conjecture by Askey, that the zeros of Jacobi polynomials $p_n = P_n^{(\alpha,\beta)}$ and $r_n = P_n^{(\gamma,\beta)}$ interlace when $\alpha < \gamma \leq \alpha + 2$, showing that the conjecture is true not only for Jacobi polynomials but also holds for Meixner, Meixner-Pollaczek, Krawtchouk and Hahn polynomials with continuously shifted parameters. The same method also provides mixed recurrence relations and interlacing results for the zeros of Al-Salam-Chihara, continuous q-ultraspherical, q-Meixner-Pollaczek and q-Laguerre polynomials of the same or adjacent degree as one of the parameters is shifted by integer values or continuously within a certain range. Numerical examples are given to illustrate cases where the zeros do not separate each other.

The polynomial inverse image method,

or how to prove results for general compact sets?

When, Where

Vilmos Totik, *University of Szeged and University of South Florida, Hungary and USA*

Primary AMS Classification: 26C05

Keywords and phrases: polynomial inverse image, lemniscates, polynomial inequalities, Christoffel functions, approximation on compact sets, universality

The talk reviews the polynomial inverse image method that has been developed in the last decade to transfer results from a segment or a circle to fairly general compact sets. The essence of the method is to apply a polynomial mapping to transfer known results on a segment or circle to lemniscates, and then to approximate general sets by lemniscates. Several of the applications of the method will also be presented, such as a sharp Bernstein inequality on general sets, a sharp Markoff inequality on several intervals, asymptotics for Christoffel functions, polynomial approximation on compact sets and extension of Lubinsky's universality to general spectra.

Abstract:

The completion of a C*-algebra under a weaker locally convex topology

When, Where

Camillo Trapani, *Universitá di Palermo*, *Italy*

Primary AMS Classification: 47L60

Keywords and phrases: Locally convex quasi C*-algebras; unbounded *-representations

Let $\mathcal{A}_0$ be a C*-algebra. Suppose that on $\mathcal{A}_0$ is defined a locally convex topology τ under which $\mathcal{A}_0$ is a locally convex *-algebra with continuous involution and *separately continuous* multiplication. The completion $\mathcal{A}$ of $\mathcal{A}_0$ under this topology has a natural structure of quasi *-algebra. Under certain *regularity conditions*, this quasi *-algebra attains a richer structure, called *locally convex quasi C*-algebra*, where some of the well known properties of C*-algebras extends in rather natural way (for instance, the functional calculus for positive elements). After a short overlook on the topic, we will present some examples and discuss, in more details, two results of representation (the first one refers to the commutative case and the second to the non commutative one) that describe completely the structure of locally convex quasi C*-algebras. Some of these results can be extended to the case where $\mathcal{A}_0$ is only a C*-normed algebra (i.e. not necessarily complete).

Extensions of positive linear functionals on a *-algebra

When, Where

Salvatore Triolo, *University of Palermo, Italy*

Primary AMS Classification: 46K05

Keywords and phrases: General theory of topological algebras with involution

We discuss a rather general strategy for constructing extensions of a nonclosable positive linear functional defined on a dense *-subalgebra $\mathfrak{A}_0$ of a topological *-algebra $\mathfrak{A}[\tau]$. The obtained results are applied to the commutative integration theory (we will show that they give rise to well-known extension of Lebesgue integral) and in noncommutative integration theory where they produce a generalized notion of integral for measurable operators.

Boundary conditions for $k - \psi$-contractive wedge maps

When, Where

Alessandro Trombetta, *University of Calabria, Italy*

Primary AMS Classification: 47H09

Keywords and phrases: Measure of noncompactness, $k - \psi$-contractive map, Fixed point index, eigenvalue

Aim of this talk is to give suitable boundary conditions under which the fixed point index of $k - \psi$-contractive wedge maps with $0 \leqq k < 1$ vanishes. The existence of eigenvalues of $k - \psi$-contractive maps is discussed.

References

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On a measure of noncompactness in topological linear spaces

When, Where

Giulio Trombetta, *University of Arcavacata di Rende, Cosenza, Italy*

Primary AMS Classification: 47H09

Keywords and phrases: Measure of noncompactness, weak measure of noncompactness, topological linear space

We introduce and study a new measure of noncompactness in the setting of Hausdorff topological linear spaces. In special cases this measure is a weak measure of noncompactness.

Spectral properties of partial*-algebras

When, Where

Francesco Tschinke, *Dipartimento di Metodi e Modelli Matematici, Palermo University, Italy*

Primary AMS Classification: 08A55

Keywords and phrases: Partial *-algebras, partial GC*-algebras, spectral properties

A further study of topological partial *-algebras is discussed, focusing the attention to some basic spectral properties. The special case of partial *-algebras of operators is examined first, in order to find sufficient hints for the study of the abstract case. The outcome consists in the selection of a class of topological partial *-algebras (partial GC*-algebras) that behave well from the spectral point of view and that allow, under certain conditions, a faithful realization as a partial O*-algebra.

References

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Vector integrals and absolutely summing operators in locally convex spaces

When, Where

Alexey Uglanov, *Saint-Petersburg State Polytechnical University, Russia*

Primary AMS Classification: 46G12

Keywords and phrases: Locally Convex Space, Vector Measure, Vector Integral, Absolutely Summing Operator

The report's topic is related to analysis on locally convex spaces (LCS). At present such analysis is developed much more weakly than analysis on Banach spaces. Meanwhile it is actual (let us remember only if the classical distributions' spaces). The author has consrtucted the theory of so-called vector integrals. A vector integral is the integral of the function which take values in an abstract LCS with respect

to (generally speaking, unbounded) the measure which take values in a vector-dual LCS. The domain of the definition of function and measure is an abstract measurable space. The definition of such integrals is given and the most important properties of the integrals are established. It should be noted the following. 1. The integral introduced is quite new object in mathematics. 2. The integral said has all natural properties of integral. 3. The property of continuty presents in our constraction. Namely, if the measure considered is real-valued measure then integral introduced is an ordinary Pettis integral; if in addition a LCS of values of integrable function is a Banach space then integral introduced is an ordinary Bochner integral. 4. The theory constructed has found many applications in various mathematical domains: stochastic analysis, infinite-dimensional partial differential equations, optimal control (including control of stochastic processes), others.

Absolutely summing maps of LCS play important part in our considerations. Therefore some necessary new results on maps indicated have been obtained; these results have independent importance too.

The work has been financially supported by the Russian Foundation for Basic Research (project 09-01-00677).

n-Approximately Weak Amenability of Banach Algebras

When, Where

Hashem Najafi, *Persian Gulf University, Iran*

Taher Yazdanpanah (speaker), *Persian Gulf University, Iran*

Primary AMS Classification: 46H20

Keywords and phrases: Banach Algebras, n-Weak Amenability, Approximate Amenability

We introduce new notions of approximate amenability for a Banach algebra. A Banach algebra A, is n-approximately weakly amenable, if every continuous derivation from A into the n-th dual space of A is approximately inner. First we examine the relation between m-approximately weak amenability and n-approximately weak

amenability for distinct m and n. Then we investigate (2n+1)-approximately weak amenability of module extension Banach algebras. Finally, we give an example of a Banach algebra that is 1-approximately weakly amenable but not 3-approximately weakly amenable.

References

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Spectral resolution for Calderon-Zygmund operators

When, Where

Vladimir Vasilyev, *Bryansk State University*, *Russia*

Primary AMS Classification: 42B20

Keywords and phrases: Calderon-Zygmund Operator, Spectral Resolution

With respect to mathematical objects the Calderon-Zygmund operators [1] have appeared in 1952 in the well-known authors' paper in Acta Mathematica. The simple case of these operators is a convolution operator treated as a principal value (singular integral). The kernel of such operator A is defined by its restriction on unit sphere S^{m-1} in $\mathbf{R}^m$, and its spectra in space of square integrable functions is defined by image of its symbol. In many interesting cases the spectra is a smooth curve in $\mathbf{C}$. It is found such operator A can be reconstructed by its spectra (up to automorphisms on S^{m-1}) with help of very familiar formula

$$A = \int\limits_{specA} \lambda G_\lambda d\lambda,$$

where G_λ are special projectors [2].

References

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My Italian Connections

When, Where

Péter Vértesi, *Rényi Mathematical Institute of the Hungarian Academy of Sciences, HUNGARY*

During the last about 25 years I had many common papers with Giuseppe Mastroianni and some of his students. This lecture tries to summarize the most interesting results of this cooperation.

Remarks on pseudoresolvents

When, Where

Florica Voicu, *Technical University of Civil Engineering Bucharest, Romania*

Mihai Voicu (speaker), *Technical University of Civil Engineering Bucharest, Romania*

Primary AMS Classification: 47A58

Keywords and phrases: Pseudoresolvent, Generator

Let E be a complex locally convex algebra, C the complex field and Λ a subset of C.

Definition 1 *A mapping $R : \Lambda \longrightarrow E$ is called pseudoresolvents if it is a solution of the resolvent equation $R(\lambda) - R(\mu) = (\mu - \lambda)R(\lambda)R(\mu)$ for all $\lambda, \mu \in \Lambda$.*

The concept of pseudoresolvent has been introduced by E. Hille and developed among others by T. Kato, F. Hirsch and M. Voicu.

We have discussed here conditions under which a pseudoresolvent is holomorphic or weakly holomorphic.

In the particular case when X is a locally convex space and $E = L(X)$ we have introduced a new notion of L_∞-type pseudoresolvent and pointed out the connection with C_0-equicontinuous semigroups.

Definition 2 *A pseudoresolvent $R : \Lambda \longrightarrow L(X)$ is called of L_∞-type if $(\omega, \infty) \subset \Lambda$ for some $\omega \in \mathbb{R}$ and*

$$\lim_{n\to\infty} nR(n)(x) = x$$

for any $x \in X$.

We have proved a sort of Trotter-Kato approximation theorems for pseudoresolvents and their generators.

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On Commutator of Positive Linear Operators

When, Where

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Let A and B be positive bounded linear operators in a Banach space ordered by a cone, with $AB \neq BA$. We will discuss some properties of the commutator of A and B, that is, of $AB - BA$. In particular, we establish sufficient conditions under which $AB - BA$ is quasinilpotent.

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